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White noise testing and model diagnostic checking for functional time series

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ABSTRACT

This paper is concerned with white noise testing and model diagnostic checking for stationary functional time series. To test for the functional white noise null hypothesis, we propose a Cramér–von Mises type test based on the functional periodogram introduced by Panaretos and Tavakoli (2013a). Using the Hilbert space approach, we derive the asymptotic distribution of the test statistic under suitable assumptions. A new block bootstrap procedure is introduced to obtain the critical values from the non-pivotal limiting distribution. Compared to existing methods, our approach is robust to the dependence within white noise and it does not involve the choices of functional principal components and lag truncation number. We employ the proposed method to check the adequacy of functional linear models and functional autoregressive models of order one by testing the uncorrelatedness of the residuals. Monte Carlo simulations are provided to demonstrate the empirical advantages of the proposed method over existing alternatives. Our method is illustrated via an application to cumulative intradaily returns.

1. Introduction

Functional data analysis (FDA) has emerged as an important area of statistics which provides convenient and informative tools for the analysis of data objects of high dimension/high resolution, and it is generally applicable to problems which are difficult to cast into a framework of scalar or vector observations. In many applications, especially if data are collected sequentially over time, it is natural to expect that the observations exhibit certain degrees of dependence. During the past decade, there is a growing body of research on parametric and nonparametric inference for dependent functional data. We refer the interested readers to the excellent monograph by Horváth and Kokoszka (2012). In this article, our interest concerns white noise testing (testing for serial correlation) for functional observations, and its application to model diagnostic checking.

In the univariate/multivariate time series context, white noise testing is a classical problem which has attracted considerable attention. There is a huge literature on the white noise testing problem and the existing tests can be roughly categorized into two types: time domain correlation-based tests and frequency domain periodogram-based tests (see e.g. Durlauf, 1991; Hong, 1996; Deo, 2000, among others). In the functional time series context, developments have been mainly devoted to the time domain based approaches. For example, Gabrys and Kokoszka (2007) proposed a portmanteau test for testing the uncorrelatedness of a sequence of functional observations. Horváth et al. (2013) proposed an independence test based on the sum of the L^2 norms of the empirical correlation functions. The validity of these tests is justified under the independent and identically distributed (i.i.d) assumption, and thus they are not robust to *dependent* white noise. In the univariate setting, the distinction between an i.i.d sequence and an uncorrelated sequence in the diagnostic checking context has been found to be important. On one hand, some commonly used nonlinear time series models, such as ARCH/GARCH models imply white noise but are dependent. On the other hand, the limiting null distributions of some commonly used test statistics, such as Box and Pierce's portmanteau test, are obtained under the i.i.d assumption, and they are no longer valid under the assumption of dependent white noise (Romano and Thombs, 1996; Lobato et al., 2002; Francq et al., 2005; Horowitz et al., 2006; Shao, 2011b). In the functional setting, this distinction is again important. For example, a functional ARCH(1) process (FARCH(1)) is a functional white noise but is dependent (Hörmann et al., 2013). In this paper, we propose a spectra-based testing procedure which is robust to the dependence within the white noise sequence. Our test is constructed based on the periodogram function recently

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introduced by Panaretos and Tavakoli (2013a) and it has nontrivial power against Pitman's local alternatives that are within a $\sqrt{T}$-neighborhood of the null hypothesis, where T denotes the sample size. Our test also avoids projecting functional objects onto a space whose dimension is held fixed in the asymptotics. Under suitable weak dependence assumptions, we show that the spectra-based test has a non-pivotal limiting distribution. To conduct inference, we introduce a novel block bootstrap procedure which is able to imitate the limiting null distribution. It is worth pointing out that when the white noise is a martingale difference sequence in a functional space, the block size in our bootstrap procedure is allowed to be one.

In statistical modeling, diagnostic checking is an integrable part of model building. A common way of testing the adequacy of the proposed time series model is by checking the assumption of white noise residuals. Systematic departure from this assumption implies the inadequacy of the fitted model. We employ the proposed white noise testing procedure to test the goodness of fit for functional linear models and functional autoregressive models of order one (FAR(1)) with uncorrelated but possibly dependent errors. Diagnostic checking in functional linear models has been studied by Chiou and Müller (2007) and Gabrys et al. (2010). The latter authors proposed two inferential tests (GHK tests hereafter) for error correlation. The main differences between our test and the GHK tests are threefold. First, our test is constructed based on the L^2 norm of the periodogram function, and it does not involve the choices of functional principal components and lag truncation number which are required in the GHK tests. Second, we do not assume any asymptotic independence under the null of the errors while the asymptotic validity of the GHK tests is established under the independence assumption. The asymptotic null distribution of our test depends on the underlying data generating process (DGP) and is no longer pivotal. We justify the validity of the proposed block bootstrap procedure when applied to the residuals in Section 3.1. Third, we employ the truncated regularization to estimate the functional linear operator, where the underlying dimension is allowed to grow slowly with the sample size. While for Gabrys et al.'s approaches, a linear model is constructed and estimated via least squares after projecting the functional objects onto a space whose dimension is fixed in the asymptotic analysis.

Furthermore, we apply the proposed method to check the adequacy of FAR(1) models. The FAR(1) model is conceptually simple as it is an extension of the univariate AR(1) model to the functional setup, yet very flexible because the autoregressive operator acts on a Hilbert space whose elements can exhibit any degree of nonlinearity. Various nonparametric and prediction methods for the FAR(1) models have been developed, and numerous applications have been found; see Besse et al. (2000), Laukaitis and Račkauskas (2002), Antoniadis and Sapatinas (2003), Fernández de Castroa et al. (2005), Kargin and Onatski (2008), among others. In spite of the wide use of the FAR(1) models, there seems no systematic methods available to check for the goodness of fit. In this paper, we shall fill in this gap by applying the spectra-based test to test the uncorrelatedness of the residuals. In contrast to the case of functional linear models, the estimation effect induced by replacing the unobservable innovations with their estimates is not asymptotically negligible due to the dependence structure of the FAR(1) models. To circumvent the difficulty, we further propose a modified block bootstrap that takes the estimation effect into account.

Finally, we point out that spectral analysis of stationary functional time series has been recently advanced by Panaretos and Tavakoli (2013a,b) and Hörmann et al. (2015). We refer to Panaretos and Tavakoli (2013a) for many interesting details on estimation and asymptotics of the spectral density operator.

The layout of the article is as follows. We introduce the spectra-based test in Section 2.1. To obtain the critical values from the limiting null distribution, we propose a block bootstrap procedure in Section 2.2. A general class of test statistics is discussed in Section 2.3. We employ the proposed method to test the goodness of fit for the functional linear models and the FAR(1) models in Section 3. Section 4 is devoted to the finite sample performance of the proposed method including simulations and an application to cumulative intradaily returns. Section 5 concludes. The proofs are postponed to Section 6 and the online supplementary material (see Appendix A).

2. White noise testing

Notation: Let $\imath = \sqrt{-1}$ be the imaginary unit. Denote by $\mathcal{I}$ a compact set of a Euclidian space. Let $\mathcal{I}^k$ be the Cartesian product of k copies of $\mathcal{I}$ with $k \in \mathbb{N}$, and $\mathcal{A} := [0, 1] \times \mathcal{I} \times \mathcal{I}$. Denote by $L^2(\mathcal{J})$ the Hilbert space of square integrable functions defined on $\mathcal{J}$, where $\mathcal{J} = \mathcal{I}^k$ or $\mathcal{A}$. For any functions $f, g \in L^2(\mathcal{J})$, the inner product between f and g is defined as $\langle f, g\rangle = \int_{\mathcal{J}} f(\tau)g(\tau)d\tau$ and $\|\cdot\|_{\mathcal{J}}$ denotes the inner product induced norm. Let $\|\cdot\| := \|\cdot\|_{\mathcal{I}}$. Assume that the random elements all come from the same probability space $(\Omega, \mathcal{F}, \mathcal{P})$. Denote by $L^p_{\mathbb{H}}$ the space of $\mathbb{H} := L^2(\mathcal{I})$ valued random variables X such that $(\mathbb{E}\|X\|^p)^{1/p} < \infty$. For any compact operator, denote by $|||\cdot|||_1$, $\|\cdot\|_{\mathcal{L}}$ and $\|\cdot\|_{\mathcal{S}}$ the nuclear norm, the uniform norm and the Hilbert–Schmidt norm respectively. Let $Re(\cdot)$ and $Im(\cdot)$ be the real part and the imaginary part of a complex number. Without ambiguity, we shall use the same symbol for operator and the kernel associated with the operator in the following discussion.

2.1. Spectra-based test

For the ease of presentation, we consider a sequence of mean-zero stationary functional time series $\{X_t(\tau)\}_{t=1}^{+\infty}$ defined on a compact set $\mathcal{I}$. With suitable modifications on the arguments, the results are expected to be extended to the situation with nonzero mean function. The lag-h autocovariance function for $\{X_t\}$ is defined as $\gamma_h(\tau_1, \tau_2) = \mathbb{E}X_t(\tau_1)X_{t-h}(\tau_2)$ with $\tau_1, \tau_2 \in \mathcal{I}$, and the corresponding autocovariance operator is given by $\gamma_h(\cdot) = \mathbb{E}\langle X_{t-h}, \cdot\rangle X_t$. Following Panaretos and Tavakoli (2013a), we define the spectral density kernel as

$$f_\omega(\tau_1, \tau_2) := \frac{1}{2\pi} \sum_{h=-\infty}^{+\infty} \gamma_h(\tau_1, \tau_2) \exp(-\imath h\omega), \quad \omega \in [-\pi, \pi]. \quad (1)$$

Given the functional observations $\{X_t\}_{t=1}^T$ where T is the sample size, we are interested in testing whether the functional time series $\{X_t\}$ has serial correlation in a functional space. In other words, we want to test the null hypothesis

$$H_0 : f_\omega(\tau_1, \tau_2) = \gamma_0(\tau_1, \tau_2)/(2\pi), \quad \text{for any } \omega \in [-\pi, \pi],$$

versus the alternative that

$$H_a : f_\omega(\tau_1, \tau_2) \neq \gamma_0(\tau_1, \tau_2)/(2\pi), \quad \text{for some } \omega \in [-\pi, \pi],$$

i.e., $\gamma_h(\tau_1, \tau_2) \neq 0$ for some lag h. To introduce the testing procedure, we define the discrete Fourier transform (DFT) of $\{X_t\}_{t=1}^T$ to be

$$\tilde{X}_\omega(\tau) := \frac{1}{\sqrt{2\pi T}} \sum_{t=1}^{T} X_t(\tau) \exp(-\imath t\omega). \quad (2)$$