Journal of Econometrics 193 (2016) 183–202

Contents lists available at ScienceDirect

Journal of Econometrics

journal homepage: www.elsevier.com/locate/jeconom

Testing for monotonicity in unobservables under unconfoundedness[☆]

Stefan Hoderlein[a], Liangjun Su[b,*], Halbert White[c], Thomas Tao Yang[d]

[a] Department of Economics, Boston College, United States
[b] School of Economics, Singapore Management University, Singapore
[c] Department of Economics, University of California, San Diego, United States
[d] Research School of Economics, Australian National University, Australia

ARTICLE INFO

Article history:
Received 9 June 2014
Received in revised form
23 October 2015
Accepted 12 February 2016
Available online 7 March 2016

JEL classification:
C12
C14
C21
C26

Keywords:
Control variables
Conditional exogeneity
Endogenous variables
Monotonicity
Nonparametrics
Nonseparable
Specification test
Unobserved heterogeneity

ABSTRACT

Monotonicity in a scalar unobservable is a common assumption when modeling heterogeneity in structural models. Among other things, it allows one to recover the underlying structural function from certain conditional quantiles of observables. Nevertheless, monotonicity is a strong assumption and in some economic applications unlikely to hold, e.g., random coefficient models. Its failure can have substantive adverse consequences, in particular inconsistency of any estimator that is based on it. Having a test for this hypothesis is hence desirable. This paper provides such a test for cross-section data. We show how to exploit an exclusion restriction together with a conditional independence assumption, which in the binary treatment literature is commonly called unconfoundedness, to construct a test. Our statistic is asymptotically normal under local alternatives and consistent against global alternatives. Monte Carlo experiments show that a suitable bootstrap procedure yields tests with reasonable level behavior and useful power. We apply our test to study the role of unobserved ability in determining Black–White wage differences and to study whether Engel curves are monotonically driven by a scalar unobservable.

1. Introduction

Global identification of structural features of interest generically involves exclusion restrictions (i.e., that certain variables do not affect the dependent variable of interest) and some form of exogeneity condition (i.e., that certain variables are stochastically orthogonal to – e.g., independent of – unobservable drivers of the dependent variable, possibly conditioned on other observables). These assumptions permit identification of such important structural features as average marginal effects or various average effects of treatment. Seminal examples are the local average treatment effects (LATE) of Imbens and Angrist (1994), the marginal treatment effects (MTE) of Heckman and Vytlacil (1999, 2005), or the control function model of Imbens and Newey (2009), (IN hereafter), to name just a few.

In addition, there may be nonparametric restrictions placed on the structural function of interest, such as separability between observable and unobservable drivers of the dependent variable ("structural separability"), or, more generally, the assumption that the dependent variable depends monotonically on a scalar unobservable ("scalar monotonicity"). Although these assumptions need not to be necessary to identify and estimate average effects of interest, when they do hold, they permit recovery of the structural function itself. This line of work dates back to Roehrig (1988). It has received a lot of attention recently; see Altonji and Matzkin (2005), AM hereafter), IN, Torgovitsky (2011), and d'Haultfoeuille and Février (2015), among others.

☆ The authors gratefully thank the co-editor Jianqing Fan, an associate editor, and two anonymous referees for their many constructive comments on the previous version of the paper. They also thank Steve Berry, Jim Heckman, Joel Horowitz, Lars Nesheim, and Maxwell Stinchcombe for helpful comments and discussion, and are indebted to Richard Blundell, Whitney Newey, and Derek Neal for providing us with their data and to Xun Lu for superlative computational assistance. The second author gratefully acknowledges the Singapore Ministry of Education for the Tier 2 Academic Research Fund (AcRF) under Grant Number MOE2012-T2-2-021.
* Corresponding author.
E-mail address: ljsu@smu.edu.sg (L. Su).

http://dx.doi.org/10.1016/j.jeconom.2016.02.015
0304-4076/

Monotonicity of a structural function in one important – yet unobservable – factor is an assumption widely invoked in economics. For instance, it is often postulated in labor economics that ability affects wages in a monotonic fashion: Other things equal, the higher the individual's ability, the higher her resulting wage. Similarly, monotonicity in unobservables has frequently been invoked in industrial organization, e.g., in the literature on production functions (see, e.g., Olley and Pakes, 1996) and the literature on auctions, where bids are monotonic functions of a scalar unobserved private valuation.

Given the wide use of monotonicity in economics and econometrics, a test for monotonicity seems desirable, not least because it has been repeatedly criticized; see, e.g., Hoderlein and Mammen (2007) or Kasy (2011). Alternatives have been suggested in the case of triangular systems (Hoderlein and Stoye, 2014), and in the treatment effects setup (Huber and Mellace, 2014). Nevertheless, to the best of our knowledge, generally applicable specification tests for monotonicity in unobservables are lacking in econometrics and statistics despite the enormous literature on nonparametric specification tests. Most closely related are specification tests in the treatment effect framework, see in particular Kitagawa (2015), but these are for a binary endogenous variable. Less closely related are tests for monotonicity in observable determinants; see, e.g., Birke and Dette (2007) and Delgado and Escanciano (2012). These latter tests are very different in structure, and generally compare a monotonized estimator with an unrestricted one. In addition, there are also tests on the structure in unobservables: Hoderlein and Mammen (2009), Lu and White (2014) and Su et al. (2015) propose convenient nonparametric tests for structural separability, but they cannot handle monotonicity. Su, Hoderlein, and White (2014, SHW hereafter) do provide a test for scalar monotonicity under a strict exogeneity assumption for large dimensional panel data models, which allows for several structural errors, but its applicability is limited by the panel data requirement.[1] Thus, our main goal and contribution here is to provide a new generally applicable test designed specifically to detect the failure of scalar monotonicity in a scalar unobservable in cross section data.

Under the null hypothesis of monotonicity of a structural function in a scalar unobservable and a conditional exogeneity assumption, we derive a testable implication that is used to construct our test statistic. We derive the asymptotic distribution of our test statistic under a sequence of Pitman local alternatives and prove its global consistency. Simulations indicate that the empirical level of our test behaves reasonably well and it has good power against non-monotonicity. The conditional exogeneity assumption holds in many important examples, including control function treatments of exogeneity, unconfoundedness assumptions as in the treatment effects literature, and generalizations of the classical proxy assumption. Note that our test does not rely on the assumption of unconditional exogeneity, and hence also works in a situation where regressors are endogenous, as long as instruments are available.

To illustrate our test, we apply our test to study the black–white earnings gap and to study consumer demand. For the former, we test the specification proposed by Neal and Johnson (1996), who include unobserved ability, A, as scalar monotonic factor, and the armed forces qualification test (AFQT) as a control variable. We fail to reject the null, providing support for Neal and Johnson's (1996) specification. That our test has power to reject monotonicity is illustrated by an analysis of Engel curves, where a scalar monotone unobservable is implausible (see (Hoderlein, 2011)). In a control function setup virtually identical to that analyzed in IN, we find that indeed the null of a scalar monotone unobservable as a description of unobserved preference heterogeneity is rejected. This suggests a demand analysis that allows for heterogeneity in a more structural fashion.

The remainder of this paper is organized as follows. In Section 2, we discuss relevant aspects of the literature on nonparametric structural estimation with scalar monotonicity, motivate our testing approach, and discuss identification under monotonicity. Based on these results, we discuss the heuristics for our test in Section 3, turning to the formal asymptotics of our estimators and tests in Sections 4 and 5. A Monte Carlo study is given in Section 6, and in Section 7 we present our two applications. Section 8 concludes. The proofs of all results are relegated to Appendix. Further technical details are contained in the online supplementary material.

2. Scalar monotonicity and test motivation

The appeal of monotonicity stems at least in part from the fact that it permits one to specify structural functions that allow for complicated interaction patterns between observables and unobservables without losing tractability. Indeed, monotonicity combined with other appropriate assumptions allows one to recover the unknown structural function from the regression quantiles. When we talk about structural models, we mean that there are random vectors Y, X and Z, and scalar random variable A, with supports $\mathcal{Y}$, $\mathcal{X}$, $\mathcal{Z}$, and $\mathcal{A}$, and only the former three being directly observable, which admit a structural relationship in the sense that there exists a measurable function $m : \mathcal{X} \times \mathcal{A} \to \mathbb{R}$ such that Y is structurally determined as

$$Y = m(X, A).$$

Note that we permit, but do not require, X and Z to be continuously distributed; either or both may have a finite or countable discrete distribution for now. As in SHW, we are interested in testing the following null hypothesis

$$\mathbb{H}_0 : m(x, \cdot) \text{ is strictly monotone for each } x \in \mathcal{X}. \tag{2.1}$$

Without loss of generality, we further restrict our attention to the case where $m(x, \cdot)$ is strictly increasing for each $x \in \mathcal{X}$ under the null; otherwise, one can always consider $-m(x, \cdot)$ if $m(x, \cdot)$ is strictly decreasing.

As SHW note, Y always has a quantile representation given X. If X is independent of A, and m is monotone in A, it allows the recovery of m. Specifically, let $G(\cdot|x)$ and $G^{-1}(\tau|x)$ denotes the conditional cumulative distribution function (CDF) and conditional τth quantile of Y given $X = x$, respectively. Then, the strict monotonicity of $m(x, \cdot)$, combined with full independence of A and X (strict exogeneity of X) and a normalization, allows the recovery of m as $m(x, a) = G^{-1}(a|x)$ for all (a, x).

Apparently, scalar monotonicity for a structural function is a strong assumption. As Hoderlein and Mammen (2007) argue, some of its implications in certain applications, such as consumer demand, may be unpalatable. In particular, monotonicity implies that the conditional rank order of individuals must be preserved under interventions to x. For example, under independence, if individual j attains the conditional median food consumption $G^{-1}(0.5|x_j)$, then he would remain at the conditional median for all other values of x.

The generic existence of the regression quantile representation, however, makes it impossible to test for monotonicity without further information. One source of such information is that

[1] See also Ghanem (2014) for related work on identification of these models.

Please cite this article in press as: Hoderlein, S., et al., Testing for monotonicity in unobservables under unconfoundedness. Journal of Econometrics (2016), http://dx.doi.org/10.1016/j.jeconom.2016.02.015