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Model selection tests for moment inequality models

Xiaoxia Shi

University of Wisconsin at Madison, 1180 Observatory Drive, Madison, WI, 53711, United States

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ABSTRACT

We propose Vuong-type tests to select between two moment inequality models based on their Kullback–Leibler distances to the true data distribution. The candidate models can be either non-overlapping or overlapping. For each case, we develop a testing procedure that has correct asymptotic size in a uniform sense despite the potential lack of point identification. We show both procedures are consistent against fixed alternatives and local alternatives converging to the null at rates arbitrarily close to $n^{-1/2}$. We demonstrate the finite-sample performance of the tests with Monte Carlo simulation of a missing data example. The tests are relatively easy to implement.

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1. Introduction

Models defined by moment inequalities (and possibly some equalities) have gained substantial popularity over recent years as researchers try to move away from ad hoc structural assumptions in various areas of economics.[1] Model selection problems in this context arise naturally when researchers consider more than one economic theory, each generating a set of moment inequalities, or when they consider different parametrizations to form the moment functions. While there is an emerging literature on parameter inference for moment inequality models, a procedure for model selection has not been available.[2] Existing model selection methods for standard models (e.g. Vuong, 1989, Kitamura, 2000, AIC, or BIC) are not applicable because moment inequality models are non-traditional in the ways discussed shortly below.

This paper provides a way to select the better model from two competing moment inequality models. We design quasi-likelihood-ratio tests for the null hypothesis that both models are equally close to the true data distribution in terms of the Kullback–Leibler (KL) divergence. When the null does not hold, the tests direct the researcher to the model that is closer to the true distribution with probability approaching one. Our tests are relatively easy to compute for two reasons. First, they use standard normal critical values. Second, although the sample criterion functions can have multiple (or even a continuum of) maximizers due to partial identification, one does not need to compute all the maximizers to implement the tests.

Moment inequality models are non-traditional in two ways. First, parameters in these models typically are not point-identified. For that reason, the maximizers of a sample criterion function do not converge to a point in the parameter space. Thus, traditional model selection methods that rely on the asymptotic normality of the maximizers do not apply. Second, moment inequality models have slackness parameters whose (pseudo-) true values may be on the boundary of the parameter space.[3] The parameter-on-the-boundary problem makes the criterion function for the original

E-mail address: xshi@ssc.wisc.edu.

[1] They have been used to model discrete games with multiple equilibria (Andrews et al., 2004, Ciliberto and Tamer, 2009), to deal with missing or interval data (Manski, 2005), to study dynamic games that are otherwise too complicated to analyze empirically (Pakes et al., 2007, Pakes, 2010) and to increase the precision of estimators in dynamic macroeconomics models (Moon and Schorfheide, 2009).

[2] A non-exhaustive list of papers on parameter inference of moment inequality models includes Chernozhukov et al. (2007), Andrews and Barwick (2012), Bugni (2010), Canay (2010), Romano and Shaikh (2010), Andrews and Guggenberger (2009), Andrews and Soares (2010) and Andrews and Shi (2013a,b).

[3] One can view the moment inequality model $Em(X_i, \theta) \geq 0$ as a moment equality model with an additional parameter a: $Em(X_i, \theta) - a = 0$. The additional parameter is the slackness parameter. The space of a is $R_+^{d_m}$. The true value of a is on the boundary of $R_+^{d_m}$ whenever a moment inequality holds as an equality under the

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model parameters non-differentiable even in the limit. The non-differentiability can occur anywhere in the original parameter space. Thus, the first-order-condition method or the standard quadratic approximation method cannot be used to derive the convergence rate of the estimators.

The first nontraditional feature prompts us to develop a new technique utilizing the stochastic equicontinuity of certain empirical processes to show the asymptotic normality of the quasi-likelihood ratio statistic and the consistency of an estimator of its asymptotic variance. The technique does not require any convergence rate of the sample maximizers. We only need a weak notion of consistency: the sample maximizers approach the pseudo-true set as the sample size goes to infinity. This technique potentially is useful to establish the asymptotic distribution of the Vuong (1989) test statistic in parametric models and moment equality models as well when the Hessian matrix of the likelihood ratio is not invertible.

The asymptotic normality and the consistency results mentioned above are sufficient for developing a valid model selection test if the asymptotic variance of the quasi-likelihood ratio statistic is bounded away from zero. The latter condition holds when the two models compared are non-overlapping in the sense defined in latter sections. When the two models are overlapping, the convergence rate of the sample maximizers is needed.

The second nontraditional feature of moment inequality models made the traditional approaches to derive convergence rate not applicable. We modify the standard quadratic approximation method and construct quadratic upper and lower bounds for the sample and population criterion functions. Combining those bounds, we show that the sample maximizers approach the pseudo-true set at $n^{-1/2}$-rate. The rate is then used to motivate an adjustment factor to the studentized quasi-likelihood ratio statistic. The adjustment factor guarantees that the adjusted test is uniformly valid for overlapping models.

The tests proposed in this paper extend the Vuong test (for maximum likelihood models) proposed in the seminal paper Vuong (1989) to models defined by moment inequalities. As such, this paper belongs to the literature that extends Vuong (1989) to various other types of models. Kitamura (2000) and Rivers and Vuong (2002) extend the Vuong test to models defined by moment *equalities*. In particular, Kitamura (2000) employs exponential tilting criterion, which is adapted to moment inequality models in the current paper. Chen et al. (2007) propose a Vuong-type procedure to select between a parametric model and a moment equality model. All these previous papers assume that the true parameters are point-identified and are in the interior of the parameter space. These assumptions are suitable for parametric models and moment equality models, but not for the moment inequality models considered in this paper. On the other hand, this paper does not make those assumption. Thus, our tests apply to point or partially identified moment inequality or equality models with or without parameter on the boundary. In the special case of non-overlapping point identified moment equality models without parameter on the boundary, our test is the same as Kitamura's (2000).

In addition to addressing the partial identification and parameter-on-the-boundary problems, another important feature distinguishing our tests from the other Vuong-type tests is that we choose the critical values based on uniform asymptotics which guarantee correct asymptotic sizes of the tests. Vuong-type tests with critical values chosen based on pointwise asymptotics may have size distortion when the candidate models are overlapping. The reason is that the pointwise asymptotic distributions of the test statistics are discontinuous in the data generating process. When the data generating process is close to the discontinuity point, the finite sample distributions of the test statistics are not well approximated by their pointwise asymptotic distributions. The poor approximation causes size distortion in finite samples (Shi, forthcoming). We adjust the test statistic in the overlapping case to take into account the discontinuity and by doing so control the asymptotic size of the tests uniformly.

An alternative to our Vuong-type framework is the Cox (1961)-type nonnested hypothesis testing framework. For a Cox-type test, the null hypothesis is that a model $\mathscr{P}$ is correctly specified and the alternative hypothesis is that an alternative model $\mathscr{Q}$ is correctly specified. Though frequently used to choose one model from multiple candidate models, Cox-type tests are intended as a procedure for model evaluation rather than model selection. A Cox-type test does not have a clear interpretation when both models are misspecified. For details on Cox-type tests, see the seminal paper by Cox (1961), the survey papers by Gourieroux and Monfort (1994) and Pesaran and Weeks (1999), generalizations to the encompassing principle by Mizon and Richard (1986), and the extension to moment equality models by Ramalho and Smith (2002). It is of interest to extend the moment encompassing principle to partially-identified moment inequality models possibly using some of the techniques developed in this paper. We leave this to a separate project.

The rest of the paper is organized as follows. Section 2 introduces the model selection problem for moment inequality models and gives a few examples. Section 3 presents preliminaries on the pseudo-distance measure and the solution to the distance-minimizing problem. Section 4 describes the tests, one for non-overlapping models and the other for overlapping models. Sections 5 and 6 establish the asymptotic size of the test for non-overlapping models and that for overlapping models, respectively. Section 7 determines the power properties of the tests. Section 8 presents Monte Carlo simulation results for a missing data example. The proofs are in the appendix.

We use $N_\delta(\theta)$ to denote a closed ball centered at θ with radius δ, $\|\cdot\|$ to denote the Euclidean norm, and "$\ll$" to denote "is absolutely continuous with respect to (w.r.t., hereafter)". We use X_i to denote an observation, $\mathcal{X}$ to denote the space on which X_i is defined. We use $\mathscr{P}$ and $\mathscr{Q}$ to denote the candidate models, and P and Q to denote generic distributions in $\mathscr{P}$ and $\mathscr{Q}$, respectively. We use μ to denote a generic true distribution on $\mathcal{X}$, which does not necessarily belong to either of the models. We use Greek letters θ and β to denote the finite-dimensional parameters in the models, Θ and B to denote the corresponding parameter spaces, and m and g to denote the moment functions.

2. Model selection problems

We consider two moment inequality/equality models $\mathscr{P} = \bigcup_{\theta\in\Theta}\mathscr{P}_\theta$ and $\mathscr{Q} = \bigcup_{\beta\in B}\mathscr{Q}_\beta$, where $\mathscr{P}_\theta$ and $\mathscr{Q}_\beta$ are the set of distributions that are consistent with the moment conditions for parameters θ and β, respectively:

$$\mathscr{P}_\theta = \left\{ P: \begin{array}{l} E_P m_j(X_i,\theta) = 0 \text{ for } j = 1,\ldots,d_p, \\ E_P m_j(X_i,\theta) \geq 0 \text{ for } j = d_p+1,\ldots,d_m \end{array} \right\}$$

$$\mathscr{Q}_\beta = \left\{ Q: \begin{array}{l} E_Q g_j(X_i,\beta) = 0 \text{ for } j = 1,\ldots,d_q, \\ E_Q g_j(X_i,\beta) \geq 0 \text{ for } j = d_q+1,\ldots,d_g \end{array} \right\}. \tag{2.1}$$

In the above equation, $\{X_i \in \mathcal{X}\}_{i=1}^n$ is a random sample generated from μ, $m = (m_1,\ldots,m_{d_p},m_{d_p+1},\ldots,m_{d_m})'$ and $g = (g_1,\ldots,g_{d_q},g_{d_q+1},\ldots,g_{d_g})'$ are R^{d_m} and R^{d_g}-valued moment functions known up to the finite-dimensional parameters θ and β, respectively, $\Theta \subset R^{d_\theta}$, $B \subset R^{d_\beta}$, and E_P denotes the expectation

true data distribution. In this example, $\{X_i\}$ is the data, m is a R^{d_m}-valued moment function and θ is a finite-dimensional parameter.

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