Journal of Econometrics 187 (2015) 154–168

Contents lists available at ScienceDirect

Journal of Econometrics

journal homepage: www.elsevier.com/locate/jeconom

Nonparametric tests for constant tail dependence with an application to energy and finance[☆]

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ARTICLE INFO

Article history:
Received 23 August 2013
Received in revised form
23 September 2014
Accepted 3 February 2015
Available online 17 February 2015

JEL classification:
C12
C14
C32
C58
G32

Keywords:
Break-point detection
Multiplier bootstrap
Tail dependence
Weak convergence

ABSTRACT

New tests for detecting structural breaks in the tail dependence of multivariate time series using the concept of tail copulas are presented. To obtain asymptotic properties, we derive a new limit result for the sequential empirical tail copula process. Moreover, consistency of both the tests and a break-point estimator are proven. We analyze the finite sample behavior of the tests by Monte Carlo simulations. Finally, and crucial from a risk management perspective, we apply the new findings to datasets from energy and financial markets.

1. Introduction

Modeling and estimating stochastic dependencies has attracted increasing attention over the last decades in various fields of applications, including mathematical finance, actuarial science or hydrology, among others. Of particular interest, especially in risk management, is a sensible quantitative description of the dependence between extreme events, commonly referred to as tail dependence; see for example Embrechts et al. (2003). A formal definition of this concept is given in Section 2.

In applications, tail dependence is often assessed by fitting a parametric copula family to the data and by subsequently extracting the tail behavior of that particular copula. Examples can be found in Breymann et al. (2003) and Malevergne and Sornette (2003), among others. Fitting the copula typically requires some sort of goodness-of-fit testing. Recent reviews on these methods are given by Genest et al. (2009) and Fermanian (2013). More robust methods to assess tail dependence are based on the assumption that the underlying copula is an extreme-value copula. The class of these copulas can be regarded as a nonparametric copula family indexed by a function on the unit simplex (Gudendorf and Segers, 2010). Since the copula is a rather general measure for stochastic dependence, the estimation techniques for both of the latter approaches are usually based on the entire available dataset (see, for instance, Genest et al. (1995), Chen and Fan (2006) for parametric families or Genest and Segers (2009) for extreme-value copulas). However, due to the fact that the center of a distribution does not contain any information about the tail behavior, these techniques might in general yield biased estimates for the tail dependence. We refer to Frahm et al. (2005) for a more elaborated discussion of this issue. In order to circumvent the problem and

☆ We are grateful to comments from the co-editor Yacine Ait-Sahalia, an associate editor, four referees, Holger Dette and Walter Krämer. This work has been supported in part by the Collaborative Research Center *Statistical Modeling of Nonlinear Dynamic Processes* (SFB 823, projects A1 and A7) of the German Research Foundation (DFG) and by the IAP research network Grant P7/06 of the Belgian government (Belgian Science Policy). Parts of this paper were written when the first author was a post-doctoral researcher at Université catholique de Louvain, Belgium.

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http://dx.doi.org/10.1016/j.jeconom.2015.02.002

to obtain estimators that are robust with respect to deviations in the center of the distribution, there are basically two important approaches: either one could extract the tail dependence from subsamples of block maximal data, for which extreme-value copulas provide a natural model (McNeil et al., 2005, Section 7.5.4), or one could rely on extreme-value techniques some of which are presented in Section 2. Applications of these procedures can be found in Breymann et al. (2003), Caillault and Guégan (2005), Jäschke et al. (2012), Jäschke (2014), among others.

Most of the aforementioned applications to time series data are based on the implicit assumption that the tail dependence remains constant over time. Whereas nonparametric testing for constancy of the whole dependence structure, as for instance measured by the copula, has recently drawn some attention in the literature (Remillard, 2010; Busetti and Harvey, 2011; Krämer and van Kampen, 2011; Bücher and Ruppert, 2013; Bücher et al., 2014; Wied et al., 2014), there does not seem to exist a unified approach to testing for constancy of the tail dependence. It is the main purpose of the present paper to fill this gap. Our proposed testing procedures are genuine extreme-value methods depending only on the dependence between the tails of the data and are hence robust with respect to potential (non-)constancy of the dependence between the centers of the distributions. In particular, the presented tests do not rely on the assumption of a constant copula throughout the sample period.

Our procedures are based on new limit results for the sequential empirical tail copula process, formally defined in Section 3.1. We derive its asymptotic distribution under the null hypothesis and propose several variants to approximate the required critical values. When restricting to the case of testing for constancy of the simple tail dependence coefficient, the limiting process can be easily transformed into a Brownian bridge. In this case, the asymptotic critical values of the tests can be obtained by direct calculations or simulations. In the more complicated case of testing for constancy of the whole extremal dependence structure as measured by the tail copula, we propose a multiplier bootstrap procedure to obtain approximate asymptotic quantiles. The finite-sample performance of all proposals is assessed in a simulation study, which reveals accurate approximations of the nominal level and reasonable power properties.

We apply our methods to two real datasets. The first application revisits a recent investigation in Jäschke (2014) on the tail dependence between WTI and Brent crude oil spot log-returns, which is based on the implicit assumption that the tail dependence remains constant over time. Our testing procedures show that this assumption cannot be rejected. The second application concerns the tail dependence between Dow Jones Industrial Average and the Nasdaq Composite time series around Black Monday on 19th of October 1987; it reveals a significant break in the tail dependence. However, our results do not show clear evidence for the hypothesis that this break takes place at the particular date of Black Monday.

The structure of the paper is as follows: in Section 2, we briefly summarize the concept of tail dependence and corresponding nonparametric estimation techniques. The new testing procedures for constancy of the tail dependence are introduced in Section 3. In particular, we derive the asymptotic distribution of the sequential empirical tail copula process, propose a multiplier bootstrap approximation of the latter and show consistency of various asymptotic tests. Additionally, we deal with the estimation of break-points in case the null hypothesis is rejected and make use of a data-adaptive process for the necessary parameter choice, common to inference methods in extreme-value theory. A comprehensive simulation study is presented in Section 4, followed by the two elaborate empirical applications in Section 5. All proofs are deferred to an Appendix A.

2. The concept of tail dependence and its nonparametric estimation

Let (X, Y) be a bivariate random vector with continuous marginal cumulative distribution functions (c.d.f.s) F and G. *Lower or upper tail dependence* concerns the tendency that extremely small or extremely large outcomes of X and Y occur simultaneously. Simple, widely used and intuitive scalar measures for these tendencies are provided by the well-established coefficients of tail dependence (TDC), defined as

$$\begin{aligned}\lambda_L &= \lim_{t\searrow 0} \mathbb{P}\{F(X) \le t \mid G(Y) \le t\},\\ \lambda_U &= \lim_{t\nearrow 1} \mathbb{P}\{F(X) \ge t \mid G(Y) \ge t\}\end{aligned} \tag{1}$$

see for instance Joe (1997), Frahm et al. (2005), among others.

It is well-known that the joint c.d.f. H of (X, Y) can be written in a unique way as

$$H(x, y) = C\{F(x), G(y)\}, \quad x, y \in \mathbb{R}, \tag{2}$$

where the copula C is a c.d.f. on $[0, 1]^2$ with uniform marginals. Elementary calculations show that the conditional probabilities in (1) can be written as

$$\lambda_L = \lim_{t\searrow 0} \frac{C(t, t)}{t}, \qquad \lambda_U = \lim_{t\searrow 0} \frac{\bar{C}(t, t)}{t},$$

where $\bar{C}$ denotes the survival copula of (X, Y). Therefore, the coefficients of tail dependence can be regarded as directional derivatives of C or $\bar{C}$ at the origin with direction $(1, 1)$. Considering different directions, we arrive at the so-called tail copulas, defined for any $(x, y) \in \mathbb{E} = [0, \infty]^2 \setminus \{(\infty, \infty)\}$ by

$$\Lambda_L(x, y) = \lim_{t\searrow 0} \frac{C(xt, yt)}{t}, \qquad \Lambda_U(x, y) = \lim_{t\searrow 0} \frac{\bar{C}(xt, yt)}{t}, \tag{3}$$

see Schmidt and Stadtmüller (2006). Note that the upper tail copula of (X, Y) is the lower tail copula of $(-X, -Y)$, whence there is no conceptual difference between upper and lower tail dependence.

Several variants of tail copulas have been proposed in the literature on multivariate extreme-value theory. For instance, $L(x, y) = x + y - \Lambda_U(x, y)$ denotes the stable tail dependence function, see, e.g., de Haan and Ferreira (2006). The function $A(t) = 1 - \Lambda_U(1 - t, t)$, which is simply the restriction of L to the unit sphere with respect to the $\|\cdot\|_1$-norm, is called Pickands dependence function, see Pickands (1981). All these variants are one-to-one and are known to characterize the extremal dependence of X and Y, see de Haan and Ferreira (2006). In the present paper we restrict ourselves to the case of tail copulas.

Nonparametric estimation of L and Λ has been addressed in Huang (1992), Drees and Huang (1998), Einmahl et al. (2006), de Haan and Ferreira (2006), Bücher and Dette (2013), Einmahl et al. (2012) for i.i.d. samples $(X_i, Y_i)_{i\in\{1,\ldots,n\}}$. For instance, in the case of lower tail copulas, the considered estimators are slight variants, differing only up to a term of uniform order $O(1/k)$, of the function

$$(x, y) \mapsto \frac{1}{k}\sum_{i=1}^{n} \mathbb{1}\Big(R_i \le kx, S_i \le ky\Big) \tag{4}$$

where R_i (resp. S_i) denotes the rank of X_i (resp. Y_i) among $X_1, \ldots, X_n$ (resp. $Y_1, \ldots, Y_n$), and where $k = k_n \to \infty$ denotes an intermediate sequence to be chosen by the statistician. Under suitable assumptions on k_n and on the speed of convergence in (3) the estimators are known to be $\sqrt{k_n}$-consistent. Additionally, under certain smoothness conditions on Λ, the corresponding process $\sqrt{k_n}(\hat{\Lambda} - \Lambda)$ converges to a Gaussian limit process.