Journal of Econometrics 187 (2015) 217–237

Contents lists available at ScienceDirect

Journal of Econometrics

journal homepage: www.elsevier.com/locate/jeconom

A test of the null of integer integration against the alternative of fractional integration

Cheol-Keun Cho, Christine Amsler, Peter Schmidt *

Michigan State University, United States

ARTICLE INFO

Article history:
Received 9 December 2013
Received in revised form
21 October 2014
Accepted 3 February 2015
Available online 3 March 2015

JEL classification:
C10
C22

Keywords:
Fractionally integrated process
Short memory
Unit root

ABSTRACT

This paper proposes a test of the null of integer integration against the alternative of fractional integration. The null of integer integration is satisfied if the series is either $I(0)$ or $I(1)$. We reject the null if the KPSS test rejects $I(0)$ and a unit root test rejects $I(1)$. We suggest a new unit root test (a lower-tail KPSS test applied to the differenced data) to use in this procedure. We provide critical values under standard asymptotics and fixed-b asymptotics. We prove the consistency of this testing procedure against $I(d)$ alternatives with $0 < d < 1$, and simulation evidence on the size and power of the test in finite samples is provided.

1. Introduction

In this paper we propose a test of the null hypothesis of "integer integration" against the alternative of fractional integration. More precisely, the null is that the series is either $I(0)$ or $I(1)$, while the alternative is that it is $I(d)$ with $0 < d < 1$. We will reject the null of integer integration in favor of the alternative of fractional integration if the KPSS test rejects the null of $I(0)$ and a unit root test rejects the null of $I(1)$. We propose a new unit root test to use as the second part of the testing procedure, which is a lower-tailed KPSS test based on first differences of the data, but other unit root tests like the ADF test could also have been used. We will call this two-part testing procedure the "Double-KPSS" test because it consists of two steps, but we stress that we treat the test as one test and evaluate its properties (consistency and finite sample size and power) as such.

The KPSS test of Kwiatkowski et al. (1992) was originally suggested as a test of the null of (short memory) stationarity against the alternative of a unit root. Conversely, standard unit root tests like the Dickey–Fuller tests, the augmented Dickey–Fuller (ADF) test of Said and Dickey (1984) or the Phillips–Perron test of Phillips and Perron (1988) were viewed as tests of the null of a unit root against the alternative of short-memory stationarity. So if the KPSS test rejected but the unit root test did not, the conclusion was that the series had a unit root. If the unit root test rejected but the KPSS test did not, the conclusion was that the series was short-memory stationary. If neither test rejected, the conclusion was that the data were not informative enough to decide whether the series was $I(0)$ or $I(1)$. However, if both tests rejected, there was in some sense a contradiction.

This apparent contradiction can be resolved by considering a wider class of processes, specifically long-memory processes. The leading example, and the one that we consider in this paper, is the $I(d)$ process (with $0 < d < 1$) of Adenstedt (1974), Granger and Joyeux (1980), and Hosking (1981). Since both the KPSS test and unit root tests have power against long-memory alternatives, the "double rejection" outcome can be taken as evidence that the series has long memory, as opposed to being either $I(0)$ or $I(1)$. This is not a novel observation. However, this paper is novel in its consideration of the double-testing procedure as a single test, and its investigation of the size and power properties of this test. In this regard, the basic observation is that if we set the nominal size of each of the two tests to 5%, the double test also has size of 5% asymptotically. For example, if the DGP is $I(0)$, then asymptotically the KPSS test will reject with probability 5% while the unit root test will reject with probability one, while if the DGP is $I(1)$ the converse will occur. So whether the data are $I(0)$ or $I(1)$, the probability of rejection of the double test is asymptotically 5%.

The practical issue to be faced is to what extent we can be reasonably sure that the double rejection outcome is due to

* Correspondence to: Department of Economics, Michigan State University, East Lansing, MI 48824, United States. Tel.: +1 517 355 8381; fax: +1 517 432 1068.
E-mail address: schmidtp@msu.edu (P. Schmidt).

http://dx.doi.org/10.1016/j.jeconom.2015.02.023
0304-4076/

fractional integration, as opposed to size distortions of the test under the $I(0)$ or $I(1)$ null. For example, Caner and Kilian (2001) and Müller (2005) have shown that the KPSS test has large size distortions if the DGP is $AR(1)$ with autoregressive coefficient near unity. Conversely, Dejong et al. (1992), Phillips and Perron (1988) and Vogelsang and Wagner (2013), among others, have found that the Dickey–Fuller test and its variants can have large size distortions, especially if the DGP is $ARIMA(0, 1, 1)$ with moving average root near (negative) unity. This does not imply that the Double-KPSS test will suffer from large size distortions in either of these cases, since the cases for which the KPSS test has large size distortions correspond to cases in which the unit root test may have low power, and conversely. However, it does argue for a careful investigation of the size and power properties of the new test in finite samples.

As noted above, the specific unit root test that we will use in this paper is a lower-tail KPSS test based on the data in differences. We considered using the KPSS unit root test suggested by Shin and Schmidt (1992) and Breitung (2002). However, as shown by Lee and Amsler (1997), the KPSS unit root test is not consistent against $I(d)$ alternatives with $1/2 < d < 1$. We also considered using the ADF test, but this test is known to have low power against $I(d)$ alternatives (e.g. Diebold and Rudebusch (1991), Hassler and Wolters (1994)), and there is also the practical consideration that it is easier to prove the consistency of our test against $I(d)$ alternatives for all d between zero and one than it is for the ADF test. In simulation, it makes little difference whether we use our new test or the ADF test. We also consider some alternative tests, including the LM tests (under normality) of the hypothesis that $d = 0$ or $d = 1$ in the class of $I(d)$ models with $0 \leq d \leq 1$. These tests have certain local optimality properties against $I(d)$ alternatives.

The consistency of the Double-KPSS test depends on the consistency of the KPSS test and of the unit root test we propose, and these in turn depend on the number of lags used in the estimation of the long-run variance going to infinity, but more slowly than sample size. Under this assumption we have a single critical value for each test (for each significance level), and we will refer to these as the "standard asymptotics" critical values. They do not depend on the kernel used to estimate the long-run variance or on the bandwidth (so long as the number of lags behaves as assumed above). However, following Kiefer and Vogelsang (2002a,b, 2005), we also consider "fixed-b asymptotics," where b, defined as the ratio of the number of lags to the sample size, is constant as the sample size grows. The fixed-b critical values depend on the kernel and on the value of b, and there is evidence in many settings that they yield tests with smaller size distortions than the critical values based on the standard asymptotics.

The main theoretical contribution of the paper is that we prove the consistency of the Double-KPSS test against $I(d)$ for all d between zero and one. For the KPSS test, this can be shown using existing results except for the case of $d = 1/2$, so we show the divergence of the statistic for $d = 1/2$. For the new unit root test, we establish its asymptotic distribution for $d = 0$, $0 < d < 1/2$ and $1/2 < d < 1$, and we show that the statistic converges to zero in probability when $d = 1/2$. Besides these theoretical results, the paper contains substantial simulation results to show the extent to which this testing procedure is likely to be useful in finite samples.

The plan of the paper is as follows. Section 2 gives the definitions and basic properties of stationary short memory, long memory and unit root processes, and explicitly states the testing procedure we propose. Section 3 gives our asymptotic results, using the standard asymptotics. The asymptotic limits of the two component tests are derived and consistency of the two-part test is proved. Section 4 presents the fixed-b asymptotics. Section 5 presents the results of simulations which explore the finite sample properties of the new test, and it also discusses alternative tests which could be used in our double-testing procedure. Section 6 summarizes and concludes. Finally, an Appendix gives some proofs and technical details.

2. Setup and assumptions

The data is assumed to be generated by the DGP:

$$y_t = \mu + \epsilon_t, \quad t = 1, 2, \ldots, T. \tag{1}$$

That is, we allow for non-zero level of the y_t series, but not trend. Allowing for trend would not change the basic principles of this paper, but it would change the asymptotics.

2.1. Null hypothesis

Under the null hypothesis $\{\epsilon_t\}_{t=1}^{\infty}$ is either a stationary short memory process or a unit root process. That is, either $\{\epsilon_t\}_{t=1}^{\infty}$ itself is a stationary short memory process or it is a cumulation of a short memory process.

Let $\{z_t\}_{t=1}^{\infty}$ be a time series with zero mean, and let $Z_t = \sum_{j=1}^{t} z_j$ be its partial sum. $\{z_t\}_{t=1}^{\infty}$ is said to be a short-memory process if it satisfies the following two conditions.

Assumption N1.

$$\sigma^2 = \lim_{T\to\infty} T^{-1} E\left(Z_T^2\right) \quad \text{exists and is nonzero.} \tag{2}$$

Assumption N2.

$$\forall r \in [0, 1], \; T^{-1/2} Z_{[rT]} \Rightarrow \sigma W(r), \tag{3}$$

where $[rT]$ denotes the integer part of rT, $\Rightarrow$ means weak convergence, and $W(r)$ is the standard Wiener process.

In addition to Assumptions N1 and N2, further regularity conditions are necessary for the consistency of HAC (heteroskedasticity and autocorrelation consistent) estimators. Examples of such conditions can be found in Andrews (1991), Newey and West (1987), DeJong and Davidson (2000), Jansson (2002), and Hansen (1992). We will implicitly assume that one or more of these sets of conditions hold, so that the HAC estimators that appear in our test statistics are consistent.

Unit root processes are the other class of DGP which belongs to the null hypothesis. A time series is said to be a unit root process if its first difference is a short memory process. Equivalently, a cumulation of a short memory process is a unit root process. That is, Z_t is a unit root process if

$$(1 - L) Z_t \equiv z_t \sim \text{short memory process.} \tag{4}$$

2.2. Alternative hypothesis

Under the alternative hypothesis, $\{\epsilon_t\}_{t=1}^{\infty}$ is a fractionally integrated process. Specifically, we will consider the alternative that ϵ_t follows an $I(d)$ process with $0 < d < 1$:

$$(1 - L)^d \epsilon_t = u_t, \quad u_t \sim \text{i.i.d Normal}(0, \sigma_u^2), \tag{5}$$

The class of $I(d)$ processes with $0 < d < \frac{1}{2}$ has been widely used in econometrics to represent long memory processes.[1] More generally, a stationary process is said to have long memory if

$$\lim_{n\to\infty} \sum_{j=-n}^{n} \gamma_j = \infty, \tag{6}$$

[1] For more comprehensive treatment of this topic, see Giraitis et al. (2012).