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Goodness-of-fit tests based on series estimators in nonparametric instrumental regression☆

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ABSTRACT

This paper proposes several tests of restricted specification in nonparametric instrumental regression. Based on series estimators, test statistics are established that allow for tests of the general model against a parametric or nonparametric specification as well as a test of exogeneity of the vector of regressors. The tests' asymptotic distributions under correct specification are derived and their consistency against any alternative model is shown. Under a sequence of local alternative hypotheses, the asymptotic distributions of the tests are derived. Moreover, uniform consistency is established over a class of alternatives whose distance to the null hypothesis shrinks appropriately as the sample size increases. A Monte Carlo study examines finite sample performance of the test statistics.

1. Introduction

While parametric instrumental variables estimators are widely used in econometrics, its nonparametric extension has not been introduced until the last decade. The study of nonparametric instrumental regression models was initiated by Florens (2003) and Newey and Powell (2003). In these models, given a scalar dependent variable Y, a vector of regressors Z, and a vector of instrumental variables W, the structural function φ satisfies

$$Y = \varphi(Z) + U \quad \text{with } \mathbb{E}[U|W] = 0 \tag{1.1}$$

for an error term U. Here, Z contains potentially endogenous entries, that is, $\mathbb{E}[U|Z]$ may not be zero. Model (1.1) does not involve the *a priori* assumption that the structural function is known up to finitely many parameters. By considering this nonparametric model, we minimize the likelihood of misspecification. On the other hand, implementing the nonparametric instrumental regression model can be challenging.

Nonparametric instrumental regression models have attracted increasing attention in the econometric literature. For example, Ai and Chen (2003), Blundell et al. (2007), Chen and Reiß (2011), Newey and Powell (2003) or Johannes and Schwarz (2010) consider sieve minimum distance estimators of φ, while Darolles et al. (2011), Hall and Horowitz (2005), Gagliardini and Scaillet (2011) or Florens et al. (2011) study penalized least squares estimators. When the methods of analysis are widened to include nonparametric techniques, one must confront two major challenges. First, identification in model (1.1) requires far stronger assumptions about the instrumental variables than for the parametric case (cf. Newey and Powell (2003)). Second, the accuracy of any estimator of φ can be low, even for large sample sizes. More precisely, Chen and Reiß (2011) showed that for a large class of joint distributions of (Z, W) only logarithmic rates of convergence can be obtained. The reason for this slow convergence is that model (1.1) leads to an inverse problem which is *ill posed* in general, that is, the solution does not depend continuously on the data.

In light of the difficulties of estimating the nonparametric function φ in model (1.1), the need for statistically justified model simplifications is paramount. We do not face an ill posed inverse

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problem if a parametric structure of φ or exogeneity of Z can be justified. If these model simplifications are not supported by the data, one might still be interested in whether a smooth solution to model (1.1) exists and if some regressors could be omitted from the structural function φ. These model simplifications have important potential since they might increase the accuracy of estimators of φ or lower the required conditions imposed on the instrumental variables to ensure identification.

In this work we present a new family of goodness-of-fit statistics which allows for several restricted specification tests of the model (1.1). Our method can be used for testing either a parametric or nonparametric specification. In addition, we perform a test of exogeneity and of dimension reduction of the vector of regressors Z, that is, whether certain regressors can be omitted from the structural function φ. By a withdrawal of regressors which are independent of the instrument, identification in the restricted model might be possible although φ is not identified in the original model (1.1).

There is a large literature concerning hypothesis testing of restricted specification of regression. In the context of conditional moment equation, Donald et al. (2003) and Tripathi and Kitamura (2003) make use of empirical likelihood methods to test parametric restrictions of the structural function. In addition, Santos (2012) allows for different hypothesis tests, such as a test of homogeneity. Based on kernel techniques, Horowitz (2006), Blundell and Horowitz (2007), and Horowitz (2011) propose test statistics in which an additional smoothing step (on the exogenous entries of Z) is carried out. Horowitz (2006) considers a parametric specification test. Blundell and Horowitz (2007) establish a consistent test of exogeneity of the vector of regressors Z, whereas Horowitz (2011) tests whether the endogenous part of Z can be omitted from φ. Gagliardini and Scaillet (2007) and Horowitz (2012) develop nonparametric specification tests in an instrumental regression model. We like to emphasize that their test cannot be applied to model (1.1) where some entries of Z might be exogenous.

Our testing procedure is entirely based on series estimation and hence is easy to implement. We use approximating functions to estimate the conditional moment restriction implied by the model (1.1) where φ is replaced by an estimator under each conjectured hypothesis. It is worth noting that by our methodology we can omit some assumptions typically found in related literature, such as smoothness conditions on the joint distribution of (Z, W). In addition, a Monte Carlo study indicates that the finite sample power of our tests exceeds that of existing tests.

The paper is organized as follows. In Section 2, we start with a simple hypothesis test, that is, whether φ coincides with a known function φ_0. We obtain the test's asymptotic distribution under the null hypothesis and its consistency against any fixed alternative model. Moreover, we judge its power by considering linear local alternatives and establish uniform consistency over a class of functions. In Sections 3–5 we consider a parametric specification test, a test of exogeneity, and a nonparametric specification test. The goodness-of-fit statistics are obtained by replacing φ_0 in the statistic of Section 2 by an appropriate estimator. In each case, the asymptotic distribution under correct specification and power statements against alternative models are derived. In Section 6, we investigate the finite sample properties of our tests by Monte Carlo simulations. All proofs can be found in the Appendix.

2. A simple hypothesis test

In this section, we propose a goodness-of-fit statistic for testing the hypothesis $H_0 : \varphi = \varphi_0$, where φ_0 is a known function, against the alternative $\varphi \neq \varphi_0$. We develop a test statistic based on $\mathcal{L}^2$ distance. As we will see in the following chapters, it is sufficient to replace φ_0 by an appropriate estimator to allow for tests of the general model against other specifications. We first give basic assumptions, then obtain the asymptotic distribution of the proposed statistic, and further discuss its power and consistency properties.

2.1. Assumptions and notation

The model revisited. The nonparametric instrumental regression model (1.1) leads to a linear operator equation. To be more precise, let us introduce the conditional expectation operator $T\phi := \mathbb{E}[\phi(Z)|W]$ mapping $\mathcal{L}_Z^2 = \{\phi : \mathbb{E}|\phi(Z)|^2 < \infty\}$ to $\mathcal{L}_W^2 = \{\psi : \mathbb{E}|\psi(W)|^2 < \infty\}$. Consequently, model (1.1) can be written as

$$g = T\varphi \tag{2.1}$$

where the function $g := \mathbb{E}[Y|W]$ belongs to $\mathcal{L}_W^2$. Throughout the paper we assume that an iid n-sample of (Y, Z, W) from the model (1.1) is available.

Assumptions. Our test statistic is based on a sequence of approximating functions $\{f_l\}_{l\geqslant 1}$ in $\mathcal{L}_W^2$. Let $\mathcal{W}$ denote the support of W and the marginal density of W by p_W. Let ν be a probability density function that is strictly positive on $\mathcal{W}$. We assume throughout the paper that $\{f_l\}_{l\geqslant 1}$ forms an orthonormal basis in $\mathcal{L}_\nu^2(\mathbb{R}^{d_w}) := \{\phi : \int \phi^2(s)\nu(s)ds < \infty\}$ where d_w denotes the dimension of W. For instance, if $\mathcal{W} \subset [a, b]$ then a natural choice of ν would be $\nu(w) = 1/(b-a)$ for $w \in [a, b]$ and zero otherwise.

Assumption 1. There exist constants $\eta_f, \eta_p \geqslant 1$ such that (i) $\sup_{l\geqslant 1} \int |f_l(s)|^4 \nu(s)ds \leqslant \eta_f$ and (ii) $\sup_{w\in\mathcal{W}}\left\{p_W(w)/\nu(w)\right\} \leqslant \eta_p$ with ν being strictly positive on $\mathcal{W}$.

Assumption 1(i) restricts the magnitude of the approximating functions $\{f_j\}_{j\geqslant 1}$ which is necessary for our proof to determine the asymptotic behavior of our test statistic. This assumption holds for sufficiently large η_f if the basis $\{f_l\}_{l\geqslant 1}$ is uniformly bounded, such as trigonometric bases. Moreover, Assumption 1(i) is satisfied by Hermite polynomials. Assumption 1(ii) is satisfied if, for instance, p_W/ν is continuous and $\mathcal{W}$ is compact.

The results derived below involve assumptions on the conditional moments of the random variables U given W gathered in the following assumption.

Assumption 2. There exists a constant $\sigma > 0$ such that $\mathbb{E}[U^4|W] \leqslant \sigma^4$.

The conditional moment condition on the error term U helps to establish the asymptotic distribution of our test statistics. The following assumption ensures identification of φ in the model (2.1).

Assumption 3. The conditional expectation operator T is nonsingular.

Under Assumption 3, the hypothesis H_0 is equivalent to $g = T\varphi_0$ which is used to construct our test statistic below. Note that the asymptotic results under null hypotheses considered in Sections 2–4 hold true even if T is singular. If Assumption 3 fails, however, our test has no power against alternative models whose structural function satisfies $\varphi = \varphi_0 + \delta$ with δ belonging to the null space of T.

We will see below that the power of our test can be increased by carrying out an additional smoothing step. Therefore, we introduce a smoothing operator L mapping $\mathcal{L}_W^2$ to $\mathcal{L}_W^2$. In contrast to the unknown conditional expectation operator T, which has to be estimated, the operator L can be chosen by the econometrician. Let L have an eigenvalue decomposition given by $\{\tau_j^{1/2}, f_j\}_{j\geqslant 1}$. We allow in this paper for a wide range of smoothing operators. In particular, L may be the identity operator, that is, no smoothing step is carried out. We only require the following condition on the operator L determined by the sequence of eigenvalues $\tau = (\tau_j)_{j\geqslant 1}$.