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Econometrics of co-jumps in high-frequency data with noise

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ABSTRACT

We establish estimation methods to determine co-jumps in multivariate high-frequency data with non-synchronous observations and market microstructure. A rate-optimal estimator of the entire quadratic covariation of an Itô-semimartingale is constructed by a locally adaptive spectral approach. Thresholding allows to disentangle the co-jump from the continuous part. We derive a feasible limit theorem for a truncated estimator of integrated covolatility which facilitates asymptotically efficient (co-)volatility estimation in the presence of jumps. A test for common jumps is presented. Simulations and an empirical application to intra-day tick-data from EUREX futures demonstrate the practical value of the approach.

1. Introduction

Last years have seen a tremendous increase in intra-day trading activities. High-frequency trading stimulated a new angle on financial modeling arousing great interest in the field of statistics of high-frequency data (HF-data). Asset prices recorded as HF-data are close to continuous-time observations and thus foster statistical inference for continuous-time price models. Demanding absence of arbitrage leads to models in which asset prices are described by semimartingales, see Delbaen and Schachermayer (1994), Imkeller and Perkowski (2013) and references therein. These include recent price models allowing for stochastic volatility and leverage. Though there is an ongoing discussion if the evolution of log-prices can be more accurately modeled by pure jump-type or continuous semimartingales, there is a broad consensus that (large) jumps occur as responses to news flow in the markets.

The efficient market hypothesis states that asset prices reflect all available information including market participants' expectations about future economic developments. Relevant news which, for instance, may come from policy announcements or macroeconomic data releases cause updates of these expectations evoking price jumps and often affect various markets and assets simultaneously. We detect such co-jumps from HF-data accounting for market microstructure and non-synchronous trading. We derive a completely data-driven method to infer on the co-jump and continuous covolatility dynamics. For portfolio and risk management, it is essential to quantify these different measures of risk and to distinguish between idiosyncratic and systematic risk. The presented approach provides access to study concerted or distinct reactions of different assets to events by quantifying and locating co-jumps. It is meant to afford deep insights into the mechanisms of default contagion and news announcements. We present a locally adaptive spectral approach to draw statistical inference on the quadratic covariation of a multi-dimensional Itô-semimartingale from discrete noisy and non-synchronous HF-data. Our methods allow to estimate the entire quadratic covariation in the presence of co-jumps as well as jump covariation and integrated covolatility (sometimes called integrated covariance) separately, both disentangled from microstructure frictions. The presented approach builds upon the spectral estimator by Bibinger and Reiß (2014) and truncation methods in the vein of Mancini (2009) and Jacod (2008).

In the one-dimensional case various estimation methods for the integrated volatility from discretely observed semimartingales with jumps have been developed. In this context, let us mention

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the important contributions by Barndorff-Nielsen and Shephard (2006), Fan and Wang (2007), Jiang and Oomen (2008), Bollerslev et al. (2008), Mancini (2009), Jacod (2008) and Corsi et al. (2010). Aït-Sahalia and Jacod (2009) have established a prominent test for the presence of jumps. An overview and an empirical comparison can be found in Theodosiou and Zikes (2011). Li (2013) is a recent important contribution on inference for jumps in noisy HF-data. In contrast to the one-dimensional case, there is scant literature on the multivariate setup yet. An important step for considering co-jumps in a multi-dimensional framework and extending truncation methods has been laid by Jacod and Todorov (2009) and Gobbi and Mancini (2012). However, their estimators are designed for non-perturbed observations only.

One main contribution of this article is to develop tractable estimators for more complex models taking market microstructure and non-synchronicity into account. Under noise perturbation the identification and localization of (co-)jumps is more challenging, since the basic principle that large returns represent (large) jumps in the efficient log-price is not valid due to the nuisance microstructure. Inference on the volatility of a continuous semimartingale under noise contamination can be pursued using smoothing techniques. Several approaches have been invented, prominent ones by Zhang (2006), Barndorff-Nielsen et al. (2008), Jacod et al. (2009) and Xiu (2010) in the one-dimensional setting and generalizations for non-synchronous multi-dimensional data by Aït-Sahalia et al. (2010), Barndorff-Nielsen et al. (2011), Park and Linton (2012), Christensen et al. (2013) and Bibinger and Reiß (2014), among others. For the one-dimensional setup Aït-Sahalia et al. (2012) have proposed a generalization of the test for jumps by Aït-Sahalia and Jacod (2009) using pre-averaging methods and exploiting the asymptotic theory by Jacod et al. (2010). A recent advance towards the estimation of the integrated covolatility under jumps has been given in Jing et al. (2013). In contrast to previous approaches, our focus is on estimating the entire quadratic covariation, jump covariation and integrated covolatility instead of concentrating only on the continuous part.

The basis for the construction of our approach is the spectral estimator by Bibinger and Reiß (2014). It relies on a locally (quasi-)parametric estimation technique in the Fourier domain. Bibinger et al. (2014) recently proved that non-synchronicity effects are asymptotically negligible in superposition with noise. In the light of this finding, and differently to preceding methods as Christensen et al. (2013) and Jing et al. (2013), we construct our estimators with equispaced blocks to average noisy observations, equally for all components. This reduces the estimator's variance. An approach combining the spectral estimator and block-wise truncation provides an estimator for integrated covolatility in the presence of (co-)jumps. We prove a feasible central limit theorem with optimal convergence rate for jumps of finite variation. Moreover, under mild smoothness assumptions on the covolatility, even allowing for jumps, the approach attains nonparametric lower bounds for the variance which have been found by Reiß (2011) and Bibinger et al. (2014). As the estimated variance is implicitly derived our method directly allows for confidence. In order to construct a rate-optimal estimator of the entire quadratic covariation and a jump covariation estimator we refine the spectral approach exploiting local estimates on shifted blocks. We establish a stable central limit theorem for the estimator of the entire quadratic covariation.

In order to derive a test for the presence of co-jumps, we adopt a strategy related to the wild bootstrap principle by Wu (1986), from Podolskji and Ziggel (2010) who have constructed a related test for jumps of one-dimensional semimartingales. Furthermore, a co-jump localization procedure in the spirit of Lee and Mykland (2008) is given. Finally, a locally adaptive thresholding strategy involving pre-estimated spot covolatilities renders an effective finite-sample approach. It accounts for intra-day volatility shapes and gives a reproducible data-driven threshold selection.

The article is arranged in six sections and an Appendix. In the next section, we introduce the statistical model and fix the notation. Theoretical results are given in Section 3, where we also carry out the construction of the estimation approach. In Section 4, we pursue the asymptotic theory for the test for co-jumps based on the wild bootstrap idea. Section 5 comes up with an implementation of the econometric estimation procedure for HF-data—adjusted to finite sample issues and discussing some practical features. In Section 6 we investigate our approach in a simulation study and show its applicability in an empirical example. Section 7 concludes. Technical proofs are postponed to the Appendix.

2. Theoretical setup

We consider prices recorded as high-frequency data from d individual assets. The evolution of underlying continuous-time log-price processes are driven by a d-dimensional Itô-semimartingale

$$\begin{aligned} X_t &= X_0 + \int_0^t b_s\,ds + \int_0^t \sigma_s\,dW_s \\ &\quad + \int_0^t \int_{\mathbb{R}^d} \kappa(\delta(s,x))(\mu-\nu)(ds,dx) \\ &\quad + \int_0^t \int_{\mathbb{R}^d} \bar{\kappa}(\delta(s,x))\mu(ds,dx) \\ &= C_t + J_t, \quad t \in \mathbb{R}_+, \end{aligned} \tag{1}$$

on a suitable filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t), \mathbb{P})$ with a right-continuous and complete filtration. The first three addends are composed to the continuous part $(C_t)_{t\ge 0}$ with W being a d-dimensional standard Wiener process and σ_t the $(d \times d')$-dimensional stochastic instantaneous volatility process. The jump part $(J_t)_{t\ge 0}$ is decomposed in a finite sum of large jumps and compensated small jumps using truncation functions $\kappa : \mathbb{R}^d \to \mathbb{R}^d$, $\bar{\kappa}(z) = z - \kappa(z)$. The Poisson random measure μ on $(\mathbb{R}_+ \times \mathbb{R}^d)$ is compensated by its intensity measure $\nu(ds, dx) = ds \otimes \lambda(dx)$ with a σ-finite measure λ on $\mathbb{R}^d$ endowed with the Borelian σ-algebra. If $\lambda(\mathbb{R}^d) = \infty$ the process is said to have infinite activity. A predictable function δ is used to shift the cut-off in time. Our notation follows Jacod (2012) and Jacod and Todorov (2009), among others, and we refer to Jacod (2012) for background information on semimartingales. The structural assumption is accomplished by the following restrictions on the characteristics of the semimartingale.

Assumption (H). The drift is a d-dimensional $(\mathcal{F}_t)$-adapted locally bounded process with $b_s = g_b\big(b_s^{(A)}, b_s^{(B)}\big)$, where $b_s^{(A)}$ is an Itô-semimartingale with locally bounded characteristics and $b_s^{(B)}$ Hölder smooth with index $\beta > 0$, i.e. $\|b_s^{(B)} - b_t^{(B)}\| \le K|s-t|^\beta$ for some constant K. The volatility σ is a $(d \times d')$-dimensional $(\mathcal{F}_t)$-adapted càdlàg process with $\sigma_s\sigma_s^\top \ge 0$ uniformly which satisfies $\sigma_s = g_\sigma\big(\sigma_s^{(A)}, \sigma_s^{(B)}\big)$, where $\sigma_s^{(A)}$ is an Itô-semimartingale with locally bounded characteristics and $\sigma_s^{(B)}$ Hölder smooth with index $\tilde{\beta} > 1/2$. The functions g_b, g_σ are continuously differentiable in both coordinates. $\sup_{\omega,x} \|\delta_\omega(s,x)\|/\gamma(x)$ is locally bounded for a deterministic non-negative function γ satisfying

$$\int_{\mathbb{R}^d} (\gamma^r(x) \wedge 1)\lambda(dx) < \infty, \tag{2}$$

for $r = 2$, or in some case for specified $r \in (0, 2]$, as stated below.

The smallest possible r such that (2) holds is sometimes called jump activity index of semimartingales and bounding the pathwise generalized Blumenthal–Getoor index. The general volatility model allows for the composition of a random semimartingale, including volatility jumps, with a function that can capture the pattern of intra-day seasonality. For the drift which gives a nuisance term of small order in the high-frequency setup the assumed