Journal of Econometrics 190 (2016) 79–99

Contents lists available at ScienceDirect

Journal of Econometrics

journal homepage: www.elsevier.com/locate/jeconom

Methods for measuring expectations and uncertainty in Markov-switching models[☆]

Francesco Bianchi[*]

Cornell University, United States
Duke University, United States
CEPR, United Kingdom
NBER, United States

ARTICLE INFO

Article history:
Received 15 April 2014
Received in revised form
23 November 2014
Accepted 28 August 2015
Available online 8 September 2015

JEL classification:
C32
C11
E31
E52
G12

Keywords:
Markov-switching VAR
DSGE
Moments
Expectations
Uncertainty
Impulse responses

ABSTRACT

I develop methods to analyze multivariate Markov-switching models. Formulas for the evolution of first and second moments are derived and then used to characterize expectations, uncertainty, impulse responses, sources of uncertainty, and welfare implications of regime changes in general equilibrium models. The methods can be used to capture the link between uncertainty and the state of the economy. Campbell's present value decomposition is generalized to allow for parameter instability. Taking into account regime changes is shown to be important for expectations, welfare, and uncertainty. All results are derived analytically and are therefore suitable for structural estimation.

1. Introduction

Since the seminal contribution of Hamilton (1989), Markov-switching models have become a popular tool to allow for parameter instability. In recent years, the univariate framework proposed by Hamilton (1989) has been extended to the multivariate case. Sims and Zha (2006) have used a Markov-switching vector autoregression (MS-VAR) to investigate the possibility of structural breaks in the conduct of monetary policy, while Sims et al. (2008) have outlined the methods for inference in this class of models. Furthermore, a growing literature has moved in the direction of modeling parameter instability in dynamic stochastic general equilibrium (DSGE) models using Markov-switching processes. While the methods to estimate multivariate Markov-switching models are by now quite well understood, regimes are often studied in isolation and the profession is still missing a framework to systematically analyze the properties of these models. This paper aims to fill this gap. I first derive a toolbox that can be used to characterize agents' expectations, model dynamics, and uncertainty in multivariate Markov-switching models. I then present a wide range of applications meant to highlight the importance of taking into account the possibility of regime changes when characterizing agents' uncertainty, the link between the macroeconomy and uncertainty, and the welfare consequences of uncertainty.

In the first part of the paper, I derive analytical laws of motion for the first and second moments of the endogenous variables. These are then combined to obtain the evolution of the covariance and auto-covariance matrices. Means and variances derived in this way take into account all sources of uncertainty, including the possibility of regime changes. I then state the conditions

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[☆] I would like to thank the editor Yacine Ait-Sahalia, two anonymous referees, Luigi Bocola, Frank Diebold, Leonardo Melosi, Juan Rubio-Ramirez, Frank Schorfheide, Allan Timmermann, and all seminar participants at the University of Pennsylvania and the NBER Summer Institute for useful comments and suggestions. Yang Yu provided excellent research assistance.

[*] Correspondence to: 464 Uris Hall, Department of Economics, Ithaca, NY 14853-7601, United States.

E-mail addresses: francesco.bianchi@cornell.edu, francesco.bianchi@duke.edu.

http://dx.doi.org/10.1016/j.jeconom.2015.08.004

under which the moments converge to finite values. Specifically, borrowing from the engineering literature, I make use of the concept of mean square stability. A process is mean square stable if its first and second moments converge to finite values as the time horizon goes to infinity. It is then straightforward to derive ergodic values for the first and second moments and, consequently, for volatilities. Mean square stability is a desirable condition to impose on a statistical process when thinking about economic applications. First, it implies that agents' expectations and uncertainty converge as the time horizon increases. Second, under the assumption of ergodicity of the Markov chain, a Markov-switching (MS) model is mean-square stable if and only if it is asymptotically covariance stationary.

I make use of these results to emphasize how MS models can be a powerful tool to characterize the evolution of agents' expectations and uncertainty. I consider a MS-DSGE model that allows for heteroskedasticity and changes in monetary policy. Once it is linearized and solved, the model returns a multivariate Markov-switching model of the kind studied by Sims and Zha (2006). As a first application, I show how to characterize the historical evolution of agents' expectations and uncertainty. At each point in time I compute the expected values and the volatilities for each of the endogenous variables at different horizons: $\mathbb{E}_t\left(Z_{t+s}\right)$ and $sd_t\left(Z_{t+s}\right) = \sqrt{\mathbb{V}_t\left(Z_{t+s}\right)}$. Expectations and uncertainty computed in this way reflect all sources of uncertainty faced by an agent in the model. Specifically, they take into account the possibility of regime changes, uncertainty around the state of the economy, uncertainty about the regime in place, and the possibility of Gaussian shocks. Therefore, they provide an accurate characterization of agents' expectations and uncertainty, based on the estimates for the model parameters and the regime probabilities.

The same formulas can be easily adapted to compute impulse responses, taking into account the possibility of regime changes. When working with models with parameter instability, two different sets of results might be of interest. First, it might be useful to understand how shocks propagate under a specific regime. In this case, the evolution of the variables of interest can be computed assuming that a specific regime is in place over the relevant horizon. However, in many other situations it might be important to take into account the possibility of regime changes. For example, a policy maker might be interested in the propagation of a shock, taking into account uncertainty about the underlying state of the economy. Alternatively, a practitioner could find it important to control for uncertainty about the future conduct of fiscal and monetary policies. In all of these cases, an impulse response can be obtained shocking the economy and then using the law of motion for first moments to project the shock into the future. The resulting impulse response automatically integrates over all possible regime paths.

A similar argument holds for uncertainty. When taking into account the possibility of regime changes measures of uncertainty can change substantially and surprising results can arise. For example, in the context of the MS-DSGE model described above, if a very volatile regime is in place today, uncertainty becomes hump shaped with respect to the time horizon. In other words, agents can be more uncertain about the short run than the long run. This is because two competing forces are at play. On the one hand, events that are further into the future are naturally harder to predict. On the other hand, in the long run the probability of still being in the high volatility regime declines. This latter mechanism also determines a decline in the upper bound for uncertainty with respect to the case in which the possibility of regime changes is ruled out: When agents are in a very volatile regime combination, they are aware that eventually the economy will move to more favorable outcomes.

In other contexts, the upper bound for uncertainty can also increase as a result of regime changes. This is because regime changes can be regarded as shocks themselves. An increase in volatility is more likely to occur when regime changes also affect the conditional steady states of the model, i.e., the values to which the state variables converge if a regime is in place for a prolonged period of time. The conditional steady states are not necessarily reached by the model, given that convergence can be slow when compared to the regime persistences. Nevertheless, they generally determine important swings in the model dynamics. This additional source of volatility cannot be detected if uncertainty is computed conditioning on a specific regime. Therefore, if an economist is interested in characterizing the effective level of uncertainty implied by an MS model, it is important to take into account the possibility of regime changes.

The same logic applies if the goal is to understand the sources of uncertainty. Some shocks might be very important under a specific regime, but much less under another one. If regime changes are ruled out when computing the variance decomposition, the importance of a specific shock might be dramatically overstated. This is because in a model subject to regime changes, it is not only the size and the contemporaneous impact of a shock that matter. A regime might be characterized by very large shocks, but such shocks may occur very infrequently or only for a very short period of time. Alternatively, it might be systematically followed by an offsetting regime that strongly mitigates the propagation of the shocks. In both cases, the overall contribution of the shocks associated with such a regime is going to be very small.

Correctly characterizing the level of uncertainty is extremely important when conducting welfare analysis in a general equilibrium model. This is because measures of medium- and long-run uncertainty change substantially when taking into account the possibility of regime changes. As a result, the importance of the regime that is in place at a particular point in time is substantially reduced. If welfare were computed assuming a regime in place for a prolonged period of time, the results could be completely misleading. In other words, it is not enough to account for the size and the contemporaneous impact of the shocks when evaluating the welfare implications of a regime. The results derived in this paper can be used to address these issues in a systematic way. Following Rotemberg and Woodford (1999), Woodford (2003), and Galí (2008), I use a period welfare loss function that depends on expected quadratic deviations of inflation and the output gap from their respective steady states. For each initial regime, these squared deviations need to be computed by integrating over all possible regime paths. Under the assumption of mean square stability, this can be done in one step by computing the discounted present value of the expected second moments as implied by the corresponding law of motion. It is worth pointing out that this way of calculating welfare takes into account uncertainty around the regime that is in place today, the current state of the economy, and the possibility of regime changes. In the long run, the second moments converge to their ergodic steady states, while the first moments converge to zero. Therefore at long horizons, welfare is determined by the ergodic variance. This is in line with standard results in the literature about welfare calculations in new-Keynesian models.

Markov-switching models can also generate interesting dynamics between uncertainty and the endogenous variables. To make this point, I simulate a bivariate MS-VAR with no Gaussian shocks. In this context, the only source of variation is represented by swings in the constant. What emerges is a model in which a variable can experience a sharp drop preceded by a sudden increase in uncertainty. At the same time, in an MS model uncertainty moves in response to the state of the economy. This is not the case in a model with fixed coefficients or in which the only source of parameter instability is due to heteroskedasticity or to shifts in the constant. The intuition for this result stems from the