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Testing for monotonicity under endogeneity
An application to the reservation wage function

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ABSTRACT

This paper develops a test for monotonicity of nonparametric regression models under endogeneity, which in its generality is novel in the literature. The test statistic, which is built upon a second order U-process, introduces 'correction terms' based on control functions that purge the endogeneity. The test has a non-standard asymptotic distribution from which asymptotic critical values can directly be derived. Furthermore, the test statistic is extended to accommodate multivariate (exogenous) regressors. Consistency against general alternatives is proved and the finite sample properties of the test are examined in a Monte Carlo experiment. The test is used to formally assess the monotonicity of the reservation wage as a declining function of elapsed unemployment duration, which has implications for underlying job search models. This relationship is difficult to measure due to the simultaneity of both variables. Results for UK data indicate that reservation wage functions do in fact not decline monotonically thereby contradicting some partial equilibrium job search models.

1. Introduction

Tests for monotonicity have been a long standing topic in the statistical and econometric literatures as applied economists have often been concerned with determining if a functional relationship between two random variables is monotonic (i.e. increasing or decreasing). In many cases, knowledge about that relationship conveys information about the properties of these variables or even of the underlying economic model itself. In practice, however, many of these functional relationships are difficult to evaluate due to simultaneity in the causal relationship, to omitted variables in the reduced-form model, or to other factors such as measurement error.

This paper develops a novel nonparametric test for monotonicity of a regression model that can be applied when the continuous regressor of interest is endogenous. I argue that such a testing framework can be relevant for various setups, for instance in labor economics and consumer theory: the functional relationship between annual hours worked and the hourly wage rate (Vella, 1993), between disposable income and expenditure on goods and services (so called Engel curves), or between the reservation wage and unemployment duration. In all three examples, standard monotonicity tests from the literature fail to be applicable due to the simultaneity of the variables of interest. On the other hand, the ability to formally test for a monotonically declining (or increasing) relationship is of interest because the outcome conveys economic implications about the underlying properties of the variables, e.g. the nature of the good in the Engel curve example. For another example, consider the case of choice of schooling as in Garen (1984): treating schooling as a continuous choice variable and ability as an omitted variable as in his paper, a further application could be to test for increasing returns to schooling. Finally, the test proposed in this paper may also be relevant for the problem of imputing missing expenditure data on non-durables as considered by Blundell et al. (2008): the authors develop an empirical strategy that allows researchers to combine information from the Consumer Expenditure Survey (CEX) and the Panel Study of Income Dynamics (PSID), where the latter is a long-run panel survey that misses expenditure data on non-durables. The strategy consists of inverting a demand for food equation estimated from CEX data. Under monotonicity of food demands these functions can be inverted to obtain a measure of non-durable consumption in the PSID. Testing this necessary condition for the Engel curves with the test at hand thus provides an opportunity to counter-check the validity of the procedure.

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The main contributions of this paper are as follows: first, I develop a nonparametric test to evaluate the monotonicity of regression models when the continuous regressor of interest is endogenous. In this generality (see below), it is the first paper within a relatively large strand of literature on monotonicity testing that formally develops a statistical test with this feature. The test is constructed on the basis of a test statistic introduced by Ghosal et al. (2000), but "correction factors" are incorporated into that statistic in order to purge the endogeneity bias: if a suitable instrument (vector) that meets a conditional mean independence assumption exists, these first-stage correction factors can be identified using standard arguments from the control function literature (e.g., Newey et al., 1999; Blundell and Powell, 2003). Moreover, subsequent use of kernel methods allows to estimate these correction terms nonparametrically and to incorporate them into the test statistic.

Second, in the supplementary material of this paper (see Appendix C), the above test statistic is formally extended to accommodate multivariate (exogenous) controls since regression models in many applications contain additional covariates researchers want to account for. More specifically, the multivariate version of the test in the supplementary material examines monotonicity of the regression model in the direction of the endogenous regressor of interest while controlling for additional (exogenous) covariates on a given set. This case has so far only been considered by Chetverikov (2012) in a non-stochastic setting and is potentially relevant in many empirical settings (see supplementary material for a motivating example, Appendix C).

Finally, as an illustration of the test, I examine the monotonicity of the functional relationship between the reservation wage and elapsed unemployment duration. While reservation wages lie at the heart of many partial and general equilibrium job search models and are viewed as a key determinant for the length of unemployment (Mortensen, 1986), elapsed unemployment durations refer to the lengths of unemployment spells at the time the reservation wage information is retrieved. The effect of unemployment duration on the reservation wage is generally ambiguous and difficult to measure because both variables are determined simultaneously if reservation wages are flexible (e.g., Kiefer and Neumann, 1979; Lancaster, 1985; Van den Berg, 1990). Moreover, despite some evidence for an overall declining reservation wage function over the course of unemployment, it is not yet well understood whether this decline is monotonic or not. Since, as will be argued in Section 5, knowledge about the monotonicity bears potential implications for underlying job search models, the paper provides a first insight into that question using self-reported hourly reservation wages and unemployment durations from the British Household Panel Survey (BHPS).

From a technical point of view, the derivation of the limiting distribution consists of two stages: the first step involves showing that, under suitable regularity and bandwidth conditions, the test statistic can be approximated by a stationary Gaussian process with continuous sample paths. While most of the proof here relies on results from Ghosal et al. (2000), the main additional step is to show that the bias arising from the estimation of the nonparametric correction terms is of smaller order asymptotically. In a second step, I prove that the excursion probability of the maximum of that Gaussian process has a non-standard Gumbel distribution (a special case of the generalized extreme value distribution) from which critical values can directly be derived. The proof technique is similar to Lee et al. (2009) in using results from Piterbarg (1996), in particular Theorem G.1 thereof (Piterbarg, 1996, p. 32). As noted in Lee et al. (2009), since the poor quality of the asymptotic approximation is one of the main issues of extreme value limiting distributions (the error declines at a rate that is logarithmic in sample size), following these authors I take advantage of a higher order analytic approximation that involves including the (known) logarithmic factor in the first-order error. This corrected distribution is closer to the actual distribution and might significantly improve the rate of convergence in finite samples. In fact, a Monte Carlo experiment in Section 4 demonstrates that this gain in power can be substantial in small samples. Finally, the test is shown to be consistent against fixed general alternatives.

Tests for monotonicity of the regression function have been a long-standing topic in the econometric and statistical literature: Bowman et al. (1998) for instance use Silverman's (1981) 'critical bandwidth' approach to construct a bootstrap test for monotonicity, while Gijbels et al. (2000) consider the length or runs of consecutive negative values of observation differences. Hall and Heckman (2000) suggest fitting straight lines through subsequent groups of consecutive points and rejecting monotonicity for too large negative values of the slopes. Other more recent tests include Durot (2003),Wang and Meyer (2011), and Birke and Neumeyer (2013). However, while for instance Durot (2003) does not impose differentiability on the nonparametric function of interest as the test of this and the other papers do, all of the aforementioned tests do require independence of the equation error and the regressor of interest, thus ruling out a wide range of economic setups that allow for dependence of those two.

A notable exception is the paper by Ellison and Ellison (2011, p. 11 ff.), who investigate the problem of detecting strategic entry deterrence in the pharmaceutical industry by examining the monotonic relationship between firms' investment levels and market size. The authors propose various monotonicity tests that also allow for the endogenous case since the incumbent's total revenue prior to patent expiration, an available proxy for market size, might be correlated with the incumbent's investment level (see Ellison and Ellison, 2011, p. 14f.). However, this case is restricted to monotonically increasing functions and it requires various monotone likelihood ratio assumptions to be satisfied. In an extension, Ellison and Ellison (2011) also mention a possible nonparametric IV approach to testing the monotonicity hypothesis under endogeneity, but no formal theory is developed.

Another test, which has recently been proposed by Chetverikov (2012), encompasses various tests from above and is based on an adaptive procedure for the construction of critical values. The adaptive choice of critical values results in a test that, unlike most of its competitors, is asymptotically non-conservative. The testing procedure has various appealing features such as consistency under conditional heteroskedasticity or the extendability to multivariate regressors, but requires those regressors to be non-stochastic, which again rules out various economic setups that are characterized by endogeneity.

A generalization of the above tests to monotonicity of nonparametric conditional distribution functions has recently been carried out by Lee et al. (2009) and Delgado and Escanciano (2012), where the former has a test statistic that is similar in nature to the one of Ghosal et al. (2000). But even though, in both examples, the null of stochastic monotonicity implies monotonicity of the regression function (if it exists), rejection of the null does clearly not imply a failure of monotonicity of that function. Finally, rather than testing for monotonicity of the regression function, Hoderlein et al. (2011) have developed a test for monotonicity of nonparametric structural regression models in a scalar unobservable (see Su et al. (2013) for the extension to panel data). Monotonicity in unobservable(s) of structural functions is often crucial as it conveys identifying power.

The paper is organized as follows: Section 2 outlines the main setup and the test statistic when a suitable control function is at hand. Section 3 derives the limiting distribution of the statistic under the null hypothesis, while Section 4 examines the finite sample properties of the estimator through a Monte Carlo experiment. Section 5 devotes attention to the reservation wage example, motivating why it is important to address endogeneity and to test for monotonicity in this context. Section 6 concludes. The proofs of the main theorems and all tables and figures are