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Binary quantile regression with local polynomial smoothing

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ABSTRACT

Manski (1975, 1985) proposed the maximum score estimator for the binary choice model under a weak conditional median restriction that converges at the rate of $n^{-1/3}$ and the standardized version has a nonstandard distribution. By imposing additional smoothness conditions, Horowitz (1992) proposed a smoothed maximum score estimator that often has large finite sample biases and is quite sensitive to the choice of smoothing parameter. In this paper we propose a novel framework that leads to a local polynomial smoothing based estimator. Our estimator possesses finite sample and asymptotic properties typically associated with the local polynomial regression. In addition, our local polynomial regression-based estimator can be extended to the panel data setting. Simulation results suggest that our estimators may offer significant improvement over the smoothed maximum score estimators.

1. Introduction

In two seminal papers Manski (1975, 1985)[1] proposed the maximum score estimator for the binary choice model under a weak conditional median restriction, which allows for very general heteroscedasticity of unknown form. Manski's work has since received a great deal of attention in the econometrics literature, and still remains a focus of active research. Manski (1975, 1985) established consistency of the maximum score estimator. Chamberlain (1986), however, showed that no $\sqrt{n}$-consistent estimator exists under Manski's assumption. In a refinement of Chamberlain's calculation, Pollard (1993) indeed showed that $n^{-1/3}$ is the best achievable rate under a mild strengthening of Manski's condition. Cavanagh (1987) and Kim and Pollard (1990) established that the maximum score estimator converges at the rate of $n^{-1/3}$ and the standardized version has a nonstandard distribution, which makes statistical inference difficult. Abrevaya and Huang (2005) further established the inconsistency of the bootstrap, complicating possible inference procedures using the maximum score estimator. Delgado et al. (2001) considered subsampling inference for the maximum score estimator and other $n^{-1/3}$-consistent estimators. Jun et al. (2014) proposed a classical (non-Bayesian) Laplace type of estimator alternative for a large class of $n^{-1/3}$-consistent estimators, including the maximum score estimator; and they further suggested a uniform inference procedure. In a context of structural estimation Fox (2008) extended the maximum score estimator to the multinomial model.

To remedy the above-mentioned drawbacks associated with maximum score estimator, in a highly influential paper, Horowitz (1992) proposed the smoothed maximum score estimator by smoothing Manski's (1975, 1985) score function under appropriate smoothness conditions. Horowitz (1992, 1993) showed that his estimator converges at least at the rate of $n^{-2/5}$ and can be arbitrarily close to $n^{-1/2}$ under these extra smoothness conditions. Furthermore, the smoothed maximum score estimator, after standardization, is shown to be asymptotically normal, which facilitates statistical inference based on standard first order asymptotic theory. In addition, Horowitz (2002) established the validity of the bootstrap for the smoothed maximum score estimator, attaining asymptotic refinements. Pollard (1993) provided complimentary results to Horowitz (1992).

While Horowitz's (1992) smoothed maximum score estimator represents a significant improvement over the maximum score estimator under these extra smoothness conditions,[2] it also has its

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[1] Manski (1987) proposed a panel version of the maximum score estimator for the binary choice panel data model with fixed effects.

[2] It is worth pointing out that the faster rate of convergence of Horowitz's estimator as well as the estimator we propose in this paper relies on these

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own serious shortcomings. As noted by Horowitz (1992), among others (e.g., Blevins and Khan (2013), Khan (2013), Iglesias (2010), Kotlyarova and Zinde-Walsh (2010), etc.), the smoothed maximum score estimator is typically associated with large finite sample bias and is quite sensitive to the choice of smoothing parameter, which makes it difficult to implement in practice. Till now, these issues have largely remained unresolved.

In this paper we develop a novel framework to systematically address the aforementioned drawbacks associated with the smoothed maximum score estimator. To this end, it is first useful to recognize that these problems are commonly associated with nonparametric kernel density and regression estimators. Indeed, as pointed out by Horowitz (1992), there is an intimate link between the smoothed maximum score estimator and the kernel nonparametric density and regression estimation. In the nonparametric estimation literature local linear and local polynomial regression estimators (e.g., Fan and Gilbels (1996)) are likely to have favorable finite sample and asymptotic properties in comparison with the kernel regression estimator when local linear or local polynomial function provides better approximation than the local constant function.[3] The local linear and polynomial regression techniques have been widely used in the semiparametric estimation literature (e.g., Fan and Gilbels (1996), Fan et al. (1995), Chen (1999), Chen and Khan (2000), Hahn et al. (2001), Heckman et al. (1997, 1998), Linton (1995, 2002), Powell and Khan (2001), etc.). In this paper we propose a novel framework which leads naturally to a local polynomial smoothing based estimator. As expected, our estimator is shown to possess the properties typically associated with the local polynomial regression estimator.

Along with the development of maximum score estimation for the cross-sectional binary choice model, there has been parallel development of distribution free estimation of the binary choice panel data model with fixed effects. Extending his maximum score estimator for the cross-sectional case, Manski (1987) proposed the maximum score estimator for the binary panel data model. Following Horowitz (1992), Charlier et al. (1995) and Kyriazidou (1997) proposed the smoothed maximum score estimator for the panel data case, and also in the same spirit, Abrevaya (2000) proposed smoothed rank estimators for a generalized fixed-effect regression model, including the maximum score and smoothed maximum score estimators as special cases. Drawbacks associated with smoothed maximum score estimators, such as large finite sample biases and sensitivity to the smoothing parameter, still plague the panel data version of the smoothed maximum score estimator, and even seem to be more serious than in the cross-sectional case. We show that our local polynomial based estimator can be extended to the panel data model, and the new estimator is shown to have properties similar to its cross-sectional counterpart.

The next section presents the identification result based on a local conditional moment condition and further proposes a local polynomial-based smoothed maximum score estimator using this identification result. Section 3 contains the asymptotic properties of the estimator. Results from Monte Carlo experiments are presented in Section 4. Section 5 concludes. The proofs of theorems are in Appendix A. Appendix B contains the extensions to the binary quantile regression model and the binary panel data model with fixed effects.

smoothness conditions. Indeed, as pointed out by Pollard (1993) that when the smoothness assumptions are violated Horowitz's estimator is dominated by a bias term.

[3] Note that the local constant regression is likely to perform better when the regression function is relatively constant. In our current context, however, the relevant regression functions, such as the conditional probability function, are not likely to be slow-changing in the region of interest, thus would be better approximated by the local linear or local polynomial regression function.

2. Smoothed maximum score estimation with local polynomial smoothing

We consider the model

$$Y = 1\left\{X'\beta + \varepsilon > 0\right\} \tag{2.1}$$

where Y is the binary dependent variable, X is a $d \times 1$ vector of independent variables, β is a corresponding vector of unknown coefficients, ε is the unobservable error term. Let β_1 denote the coefficient of the first component of X, X_1, which is assumed to be continuously distributed conditional on $\tilde{X}$, the remaining components of X. Following Horowitz (1992), we set $|\beta_1| = 1$ for scale normalization.

Under the assumption that the conditional median of ε given X is zero, i.e, $\text{med}(\varepsilon|x) = 0$, Manski (1975, 1985) proposed the maximum score estimator for β, any value b that solves the problem

$$\max_{\tilde{b}} S_M^*(b) = \sum_{i=1}^{n} (Y_i - 0.5)\, 1\left\{X_i'b > 0\right\}$$

where $\{X_i, Y_i\}_{i=1}^n$ is a random of observations generated from Model (2.1). Cavanagh (1987) and Kim and Pollard (1990) established that the maximum score estimator converges at the rate of $n^{-1/3}$ and the standardized version has a nonstandard distribution. Recognizing that the slow rate of convergence and complicated nature of the limiting distribution of the maximum score estimator are largely due to the discontinuity of the objective function S_M^*, with strengthening of smoothness conditions Horowitz (1992) proposed the smoothed maximum score estimator by replacing S_M^* with a smoothed version. Specifically, Horowitz's (1992) estimator is defined as any solution to the problem

$$\max_{b} S_H(b) = \sum_{i=1}^{n} (Y_i - 0.5)\, K\left(\frac{X_i'b}{h}\right)$$

where K is a smoothed indicator function, usually obtained by integrating a kernel density function, and h is a bandwidth parameter converging to zero as the sample size increases. While the smoothed maximum score estimator, which converges at a faster rate and is asymptotically normal under the extra smoothness conditions, represents a significant improvement over the maximum score estimator, it also has its own serious shortcomings. It is typically associated with large finite sample biases and is quite sensitive to the choice of smoothing parameter, which makes it difficult to implement in practice. To better understand the finite sample and asymptotic properties of the smoothed maximum score estimator, it is instructive to examine its close link to nonparametric kernel regression. Note that smoothed maximum score estimator $\hat{\beta}_{SMC}$ satisfies the estimating equations

$$\sum_{i=1}^{n} (Y_i - 0.5)\, \frac{1}{h} k\left(\frac{X_i'b}{h}\right) \tilde{X}_i = 0$$

which can be formulated equivalently as

$$E_n\left[(Y - 0.5)\, \tilde{X} | X'b = 0\right] = 0 \tag{2.2}$$

where

$$E_n\left[(Y - 0.5)\, \tilde{X} | X'b = 0\right] = \frac{\sum_{i=1}^{n} (Y_i - 0.5)\, \frac{1}{h} k\left(\frac{X_i'b}{h}\right) \tilde{X}_i}{\sum_{i=1}^{n} \frac{1}{h} k\left(\frac{X_i'b}{h}\right)}$$

is the common nonparametric kernel regression estimator for $E\left[(Y - 0.5)\, \tilde{X} | X'b = 0\right]$; namely, the first order conditions corresponding to the smoothed maximum score estimator can be