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Adaptive estimation of the threshold point in threshold regression

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A B S T R A C T

This paper studies semiparametric efficient estimation of the threshold point in threshold regression. The classical literature of semiparametric efficient estimation rests on the fact that the maximum likelihood estimator is efficient in any parametric submodel for a large class of loss functions. However, in threshold regression, the maximum likelihood estimator is not efficient, while the Bayes estimators are efficient and different loss functions induce different efficient estimators. For an additively separable loss function that separates the efficiency problem of the threshold point from that of other parameters, we show that the semiparametric and parametric efficiency risk bounds coincide. Then we design a semiparametric empirical Bayes estimator to achieve this bound. In consequence, the threshold point can be adaptively estimated even under conditional moment restrictions. We also provide a valid confidence interval called the nonparametric posterior interval for the threshold point. Simulation studies show that the semiparametric empirical Bayes approach is substantially better than existing methods. To illustrate our procedure in practice, we apply it to an economic growth model for detecting different growth patterns.
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1. Introduction

The linear regression model has played a prominent role in econometric analysis. One important limitation of the linear regression model is that different groups of entities may have different behaviors in a specific economic problem. For example, Durlauf and Johnson (1995) show that rich countries and poor countries have different growth patterns. The question is how to separate these two groups of countries and estimate their respective growth paths. The threshold regression (TR) model introduced by Tong (1978, 1983) and Tong and Lim (1980) is designed to answer such a question; see Tong (1990, 2011) for a summary of the TR literature in statistics and Hansen (2011) in econometrics. The typical setup of TR models is as follows:

$$y = \begin{cases} x'\beta_1 + \sigma_1 e, & q \le \gamma; \\ x'\beta_2 + \sigma_2 e, & q > \gamma, \end{cases} \tag{1}$$

where q is the threshold variable used to split the sample with pdf $f_q(\cdot)$ and cdf $F_q(\cdot)$, γ is the unknown threshold point, $x \in \mathbb{R}^k$ includes characteristics, $\beta \equiv (\beta_1', \beta_2')' \in \mathbb{R}^{2k}$ and $\sigma \equiv (\sigma_1, \sigma_2)'$ are parameters in the mean and variance of the two groups. We assume that x does not include the intercept, discrete regressors or q; these cases can be easily adapted in the following discussion. We also set $E[e^2] = 1$ as a normalization of the error variance and allow for conditional heteroskedasticity. The usual conditional moment restriction is

$$E\left[e|x, q\right] = 0. \tag{2}$$

There are two asymptotic frameworks for statistical inferences on γ. The first is introduced by Chan (1993) in a nonlinear time series context, where $\left(\beta_1', \sigma_1\right)' - \left(\beta_2', \sigma_2\right)'$ is a fixed constant. The second is introduced by Hansen (2000), where no threshold effect on variance exists and the threshold effect in mean diminishes asymptotically. This paper follows the discontinuous framework of Chan (1993) with i.i.d. data.

Both Chan (1993) and Hansen (2000) use the least squares estimator (LSE) to estimate γ, but the problem of semiparametric efficient estimation of γ has not been studied. The difficulty lies in the fact that parameters considered in most existing literature of semiparametric efficiency are regular, while γ is not a regular parameter. For a regular parameter, the maximum likelihood

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estimator (MLE) is asymptotically normal and efficient in any parametric submodel for a large class of loss functions. As a result, the efficiency of an estimator is indicated by its asymptotic variance. This is the starting point of finding semiparametric efficiency bounds. As the semiparametric problem is not easier than any parametric subproblem, the semiparametric efficiency bound is defined as the supremum of asymptotic variances of the MLEs among all submodels. In threshold regression, Yu (2012), inspired by the literature on boundary estimation such as Hirano and Porter (2003) and Chernozhukov and Hong (2004), shows that the MLE is not efficient for γ, while the Bayes estimators are efficient. Furthermore, different loss functions induce different efficient estimators. This makes the techniques for finding semiparametric efficient estimators in the existing literature not applicable.

In this paper, we solve the semiparametric efficiency problem of γ in two steps. First, in Section 2, we separate the efficiency problem of γ from that of other regular parameters by using an additively separable loss function. Given any such loss function, we show that the risk of the Bayes estimator of γ in any parametric submodel is the same as that when the true conditional density $f_{e|x,q}$ is known. Therefore, the semiparametric efficiency risk bound of γ for a given loss function is the risk in the parametric model, and the conditional moment restriction (2) does not lose any information from the completely known $f_{e|x,q}$ case. Second, in Section 3, we use a semiparametric empirical Bayes (SEB) approach to find an estimator of γ that achieves the efficiency risk bound. The SEB estimator (SEBE) is adaptive in the sense that the risk in the parametric case can be reached even if $f_{e|x,q}$ is not exactly known. It should be pointed out that all Bayes procedures in this paper are evaluated by classical efficiency criteria; in other words, the randomness is confined to the data and does not include parameters.

Although the SEBE has the same asymptotic risk as the parametric Bayes estimator, it is less susceptible to misspecification because no parametric specification is needed for the distribution of e. γ can be identified as in the correctly specified parametric model as long as (2) is imposed. Also, the SEBE avoids the Diaconis and Freedman (1986a,b)'s inconsistency problem by estimating the nuisance density $f_{e|x,q}$ rather than imposing a Dirichlet prior on it. The literature discussing how to avoid the Diaconis and Freedman's problem in the Bayesian framework all concentrates on regular models.

A corollary of the SEB method is to provide a valid confidence interval (CI) for γ. The CI construction for γ is unsolved in Chan (1993) and reconsidered in Hansen (2000). In Section 4, we discuss the difficulties in the previous papers and propose an alternative valid CI for γ—the nonparametric posterior interval (NPI). Section 5 includes some simplification and extension of the SEB approach to increase its applicability and to improve its finite-sample performance. The simulation results in Section 6 show that the SEBE has a lower risk and the NPI has better coverage and length properties than the existing methods. Section 7 applies the SEB method to an economic growth model and Section 8 concludes. All regularity conditions, proofs and tables in simulations and the application are given in Appendices A–C, respectively. Certain technical materials of the paper are collected in supplementary materials (see Appendix D).[1] Notations: the Euclidean norm of a vector $x \in \mathbb{R}^k$ is denoted as $\|x\|$, and C or C with a subscript is used as a generic positive constant, which need not be the same in each occurrence.

[1] The code for simulations and application and the supplementary materials are available at http://homes.eco.auckland.ac.nz/pyu013/research.html.

2. Semiparametric efficiency risk bound

We first recall the parametric results of Yu (2012) in Section 2.1. We then show that the semiparametric bound is the same as the parametric bound using a simple example and provide some intuition for this adaptive result in Section 2.2. At the end of Section 2.2, we also discuss a technical assumption on the loss function in the semiparametric efficiency risk bound derivation.

2.1. Parametric efficient estimation

The main results of Yu (2012) are that the Bayes estimator (BE) is more efficient than the MLE for estimating γ, the threshold point. Suppose $f_{e|x,q}$ is known as $f_{e|x,q}(e|x, q; \alpha)$, where $\alpha \in \mathbb{R}^{d_\alpha}$ is some nuisance parameter affecting the shape of the error distribution. Assume further that the loss function is additively separable on regular parameters and the nonregular parameter γ; that is, $l(\theta) = l(\underline{\theta}, \gamma) = l_1(\underline{\theta}) + l_2(\gamma)$, where $\theta = \left(\underline{\theta}', \gamma\right)'$, $\underline{\theta} = \left(\beta', \sigma', \alpha'\right)'$, and l_1 is bowl-shaped.[2] This assumption is important for the semiparametric efficient estimation of γ, as it separates the efficiency problem of γ from that of the regular parameters. Such an assumption is motivated by the sequential estimation of γ and $\underline{\theta}$. Usually, a profiled procedure is used to estimate γ first, and then estimate $\underline{\theta}$ as if γ were known; see, e.g., Hansen (2000) and Yu (2012). It is reasonable to impose a loss function on each of these two steps without interactions.

Under regularity conditions specified in Yu (2012), the BE $\left(\widehat{\underline{\theta}}_{BE}', \widehat{\gamma}_{BE}\right)'$ based on l is most efficient in the locally asymptotically minimax (LAM) sense, and the asymptotic distribution is

$$\begin{aligned} &\sqrt{n}\left(\widehat{\underline{\theta}}_{BE} - \underline{\theta}_0\right) \xrightarrow{d} Z_{\underline{\theta}} \sim N\left(0, \mathcal{I}_{\underline{\theta}_0}^{-1}\right), \\ &n\left(\widehat{\gamma}_{BE} - \gamma_0\right) \xrightarrow{d} Z_\gamma = \arg\min_t \int_{\mathbb{R}} l_2(t - v)\, p_2^*(v) dv, \end{aligned} \tag{3}$$

where $\mathcal{I}_{\underline{\theta}_0}$ is the information matrix of $\underline{\theta}$, $p_2^*(v) = \frac{\exp\{D(v)\}}{\int_{\mathbb{R}} \exp\{D(\overline{v})\} d\overline{v}}$ is the normalized asymptotic posterior of γ, and these two asymptotic distributions are independent. Note that $\widehat{\underline{\theta}}_{BE}$ has the same asymptotic distribution as the MLE. The $D(v)$ in $p_2^*(v)$ is a compound Poisson process defined as

$$D(v) = \begin{cases} \sum_{i=1}^{N_1(|v|)} z_{1i}, & \text{if } v \le 0; \\ \sum_{i=1}^{N_2(v)} z_{2i}, & \text{if } v > 0; \end{cases} \tag{4}$$

which is cadlag with $D(0) = 0$, where all z_{1i}, z_{2i}, $i = 1, 2, \ldots, N_1(\cdot)$ and $N_2(\cdot)$ are mutually independent of each other, $N_\ell(\cdot), \ell = 1, 2$, is a Poisson process with intensity $f_q(\gamma_0)$, z_{1i} follows the limiting conditional distribution of

$$\overline{z}_{1i} \equiv \ln \frac{\frac{\sigma_{10}}{\sigma_{20}} f_{e|x,q}\left(\left.\frac{\sigma_{10} e_i + x_i'(\beta_{10} - \beta_{20})}{\sigma_{20}} \right| x_i, q_i; \alpha_0\right)}{f_{e|x,q}(e_i | x_i, q_i; \alpha_0)}$$

given $\gamma_0 + \Delta < q_i \le \gamma_0$, $\Delta < 0$ with $\Delta \uparrow 0$ and is denoted as $\overline{z}_{1i} | (q_i = \gamma_0-)$, and z_{2i} follows the limiting conditional distribution of

$$\overline{z}_{2i} \equiv \ln \frac{\frac{\sigma_{20}}{\sigma_{10}} f_{e|x,q}\left(\left.\frac{\sigma_{20} e_i - x_i'(\beta_{10} - \beta_{20})}{\sigma_{10}} \right| x_i, q_i; \alpha_0\right)}{f_{e|x,q}(e_i | x_i, q_i; \alpha_0)}$$

[2] A function is defined to be bowl-shaped if the sublevel sets $\{x : l(x) \le C\}$ are convex and symmetric about the origin.

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