Journal of Econometrics 189 (2015) 229–244

Contents lists available at ScienceDirect

Journal of Econometrics

journal homepage: www.elsevier.com/locate/jeconom

Identification and estimation of games with incomplete information using excluded regressors[☆]

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ARTICLE INFO

Article history:
Received 7 March 2013
Received in revised form
16 June 2014
Accepted 23 October 2014
Available online 17 July 2015

JEL classification:
C14
C51
D43

Keywords:
Games with incomplete information
Excluded regressors
Nonparametric identification
Semiparametric estimation
Multiple equilibria

ABSTRACT

We show structural components in binary games with incomplete information are nonparametrically identified using variation in player-specific excluded regressors. An excluded regressor for a player i is a sufficiently varying state variable that does not affect other players' utility and is additively separable from other components in i's payoff. Such excluded regressors arise in various empirical contexts. Our identification method is constructive, and provides a basis for nonparametric estimators. For a semiparametric model with linear payoffs, we propose root-N consistent and asymptotically normal estimators for players' payoffs. We also discuss extension to the case with multiple Bayesian Nash equilibria in the data-generating process without assuming equilibrium selection rules.

1. Introduction

We show identification of, and provide estimators for, a class of static game models where players simultaneously choose between binary actions based on imperfect information about each other's payoffs. Such models are widely applied in industrial organization. Examples include firms' entry decisions in Seim (2006), decisions on the timing of commercials by radio stations in Sweeting (2009), firms' choices of capital investment strategies in Aradillas-Lopez (2010) and interactions between equity analysts in Bajari et al. (2010a).

• Preview of our method

We introduce a new approach to identify and estimate such models by assuming players' payoffs depend on a vector of "excluded regressors". An excluded regressor associated with a given player i is a state variable that does not enter payoffs of the other players, and is additively separable from other components in i's payoff. We show that if excluded regressors are orthogonal to players' private information given other observed states, then interaction effects between players and marginal effects of excluded regressors on players' payoffs can be nonparametrically identified. If in addition the excluded regressors vary sufficiently relative to the support of private information, then the distribution of payoffs for all players is also nonparametrically identified. No distributional assumption on the private information is necessary for these results. Our identification proofs are constructive, so consistent nonparametric estimators can be readily based on them.

We provide several examples of economic models where excluded regressors arise. For example, consider an entry game where profit maximizing firms decide simultaneously whether to enter a market or not, knowing that after entry they will compete through their choices of quantities. The fixed cost of entry for a firm i does not affect its perception of the interaction effects with other firms, or the difference between its monopoly profits (when i is the sole entrant) and its oligopoly profits (when i is one of the multiple entrants). This is precisely the independence restriction required of excluded regressors.

The intuition for this result is that fixed costs drop out of the first-order conditions in the profit maximization problems both for a monopolist and for oligopolists engaging in Cournot competition.

☆ We thank Arie Beresteanu, Aureo de Paula, Stefan Hoderlein, Zhongjian Lin, Pedro Mira, Jean-Marc Robin, Frank Schorfheide, Steven Stern and Kenneth Wolpin for helpful discussions. We also thank the editor and two anonymous referees and seminar participants at Asian Econometric Society Summer Meeting 2011, Brown U, Duke U, London School of Economics, Rice U, Penn State, U British Columbia, U Connecticut, U Pennsylvania, U Pittsburgh and U Rochester for feedbacks and comments. All errors are our own.

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http://dx.doi.org/10.1016/j.jeconom.2014.10.014
0304-4076/

As a result, while fixed costs affect the decision to enter, they do not otherwise affect the quantities supplied by any of the players. In general, only some components of a firm's fixed cost are likely to be observable by econometricians, while the remaining, unobserved part of the fixed cost is private information known only to the firm. Our method can be applied in this case, with the observed component of fixed costs playing the role of excluded regressors. Other excluded regressor conditions that contribute to identification (such as conditional independence from private information and sufficient variation relative to the conditional support of private information given observed states) also have economic interpretations in the context of entry games. We discuss those interpretations in greater detail in Section 4.

We also discuss how to extend our approach to obtain point identification when there are multiple Bayesian Nash equilibria (BNE) in the data generating process (DGP), without imposing restrictions on the distribution of private information or assuming knowledge of the equilibrium selection mechanism. In this case, identification can be obtained as before, but only using variation of excluded regressors within a set of states where the choices observed in the data are rationalized by a single BNE. We build on De Paula and Tang (2012), showing that if players' private information is independent conditional on observed states then the players' actions are correlated given these states if and only if choices observed under those states are rationalized by multiple BNE. Thus one can infer from the data whether a given set of states have a single BNE, and base identification only on those such states.

• Relation to existing literature

This paper contributes to the existing literature on structural estimation of Bayesian games in several ways. First, we identify the complete structure of the model under a set of mild nonparametric restrictions on primitives. Bajari et al. (2010a) identify the players' payoffs in a general model of multinomial choices, without assuming any particular functional form for payoffs, but they require the researcher to have full knowledge of the distribution of private information. In comparison, we allow the distribution of private information to be unknown and nonparametric. We instead obtain identification by imposing economically motivated exclusion restrictions.

Sweeting (2009) proposes a maximum-likelihood estimator where players' payoffs are linear indices and the distribution of players' private information is known to belong to a parametric family. Aradillas-Lopez (2010) estimates a model with linear payoffs where players' private information is independent of states observed in the data. He also proposes a corresponding root-N consistent estimator by extending the semi-parametric estimator for single-agent decision models in Klein and Spady (1993) to the game-theoretic setup. Tang (2010) shows identification of a similar model to Aradillas-Lopez (2010), with the independence between private information and observed states replaced by a weaker median independence assumption. He proposes consistent estimators for linear coefficients without establishing rates of convergence. Wan and Xu (2014) consider a class of Bayesian games with index utilities under median restrictions. They allow players' private information to be positively correlated in a particular way, and focus on a class of monotone, threshold crossing pure-strategy equilibria. Unlike these papers, we show non-parametric identification of players' payoffs, with no functional forms imposed. We also allow private information to be correlated with non-excluded regressors in unrestricted ways, though we do require them to be independent across players conditional on observable states.

The second contribution of our paper is to propose consistent and asymptotically normal estimators for semi-parametric models with linear payoffs and heteroskedastic private information. If in addition the support of private information and excluded regressors are bounded, our estimators for the linear coefficients in baseline payoffs attain the parametric rate. If these supports are unbounded, then the rate of convergence of estimators for these coefficients can be parametric or slower, depending on properties of the tails of the distribution of the excluded regressors. The estimators for interaction effects converge at the parametric rate regardless of the boundedness of these supports.[1]

When multiple BNE exist in the data, a player's choice probability is an unknown mixture of the choice probabilities implied under each single equilibrium. Hence, in that case the structural link between the distribution of actions and the model primitives is unknown. The existing literature offers several solutions to this multiple BNE issue. These include parameterizing the equilibrium selection mechanism; maintaining that only a single BNE is followed by the players (Bajari et al. (2010a)); introducing sufficient restrictions on primitives to ensure that the system characterizing the equilibria has a unique solution (Aradillas-Lopez (2010)); or obtaining partial identification results (Beresteanu et al. (2011)).[2] For a model of Bayesian games with linear payoffs and correlated bivariate normal private information, Xu (2014) addressed the multiplicity issue by focusing on states for which the model only admits a unique, monotone equilibrium. Xu (2014) showed that with these parametric assumptions, a subset of these states can be inferred using the observed distribution of choices, which can then be used for estimating parameters in payoffs. While different, as explained above our approach for dealing with multiple BNE is based on a similar idea.

Our use of excluded regressors is related to the use of "special regressors" in single-agent, qualitative response models (see Dong and Lewbel (2015) and Lewbel (2014) for surveys of special regressor estimators). Lewbel (1998, 2000) studied nonparametric identification and estimation of transformation, binary, ordered, and multinomial choice models using a special regressor that is additively separable from other components in decision-makers' payoffs, is independent from unobserved heterogeneity conditional on other covariates, and has a large support. Magnac and Maurin (2008) study partial identification of the model when the special regressor is discrete or measured within intervals. Magnac and Maurin (2007) remove the large support requirement on the special regressor in such a model, and replace it with an alternative tail symmetry restriction on the distribution of latent utility. Lewbel et al. (2011) estimate features of willingness-to-pay models with a special regressor constructed by experimental design. Berry and Haile (2009, 2014) extend the use of special regressors to nonparametrically identify the market demand for differentiated goods.

Special regressors and exclusion restrictions have also been used for estimating social interaction models and games with complete information. Brock and Durlauf (2007) and Blume et al. (2010) used exclusion restrictions of instruments to identify a linear model of social interactions. Bajari et al. (2010b) used exclusion restrictions and identification at infinity, while Tamer (2003) and Ciliberto and Tamer (2009) used some special regressors to identify games with complete information. Our identification differs fundamentally from that of complete information games because in our

1 Khan and Nekipelov (2011) studied the Fisher information for the interaction parameter in a similar model of Bayesian games, where private information is correlated across players, but the baseline payoff functions are assumed known. Unlike our paper, the goal of their paper is not to jointly identify and estimate the full structure of the model. Rather, they focus on showing that the Fisher information of the interaction parameter is positive while assuming the functional form of the rest of the payoffs as well as the equilibrium selection mechanism are known.

2 Aguirregabiria and Mira (2005) proposed an estimator for payoff parameters in games with multiple equilibria where identification is assumed. Their estimator combines a pseudo-maximum likelihood procedure with an algorithm that searches globally over the space of possible combinations of multiple equilibria in the data.