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Regression-based analysis of cointegration systems

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ABSTRACT

Two estimation procedures dominate the cointegration literature: Johansen's maximum likelihood inference on vector autoregressive error correction models and estimation of Phillips' triangular forms. This latter methodology is essentially semiparametric, focusing on estimating long run parameters by means of cointegrating regressions. However, it is less used in practice than Johansen's approach, since its implementation requires prior knowledge of features such as the cointegrating rank and an appropriate set of non-cointegrated regressors. In this paper we develop a simple and automatic procedure (based on unit root and regression-based cointegration testing) which, without imposing a parametric specification for the short run components of the model, provides an estimator of the cointegrating rank and data-based just-identifying conditions for the cointegrating parameters which lead to a Phillips' triangular form. A Monte Carlo analysis of the properties of the estimator and an empirical application are also provided.

1. Introduction

Cointegration has been one of the main workhorses of time series econometrics in the last two decades and, even if the literature is somewhat mature, it still attracts substantial attention from both theoretical and empirical perspectives (see, e.g., Hoover et al., 2008, or Johansen, 2010). Two approaches to the estimation of cointegration systems appear to be dominant. The first, developed by Johansen (1988, 1991) and Ahn and Reinsel (1990), focuses on maximum likelihood inference on vector autoregressive (VAR) error correction models. This approach has been the most popular in practice, mainly because it both provides an estimator of the cointegrating rank and leads to empirical (data-based) just-identifying restrictions from which estimators of the cointegrating vectors can be easily obtained. Additionally, it offers estimators of the short run parameters and a neat hypothesis testing procedure, where given economic theories can be checked. The second dominant strategy focuses on estimation of the so-called Phillips' triangular form (Phillips, 1991a). This approach, which relates directly to the simultaneous equations models methodology, consists of specifying the cointegrating relations by a set of reduced form regression equations from which estimation of structural equations (those with economic meaning) can be derived (see Saikkonen, 1993). Within this setting, different estimation methods have been proposed and, noticeably, it has been shown that parametric assumptions on the short run components do not lead to gains in asymptotic efficiency in the estimation of cointegrating vectors (see Phillips and Hansen, 1990; Phillips, 1991b). Pesaran and Shin (2002) provide a comparison of both methods.

In contrast to Johansen's approach, Phillips' methodology is essentially semiparametric, focusing on the long run components of the model and taking an agnostic approach about the short run dynamics, which, in any case, once parameterized, can be subsequently estimated if desired. While this appears to be an attractive feature (compared to a fully parametric approach), there are several limitations associated to Phillips' methodology. First, the procedure takes the cointegrating rank as given. Moreover, based on this rank, the vector of observables is decomposed into two subvectors corresponding to dependent variables and regressors in particular cointegrating regressions. Specifically, the number of dependent variables should be the same as the cointegrating rank, and the rest of the variables (regressors) must not be cointegrated, otherwise they would be perfectly correlated asymptotically. In short, Phillips' approach imposes a priori identification conditions on the cointegrating parameters, and this leads to uncertainty on how to act in practice. As a result, in comparison to Johansen's

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approach, this methodology has been hardly used in applied work, except in the single-equation framework, where an extensive literature on testing for cointegration exists (see, e.g. Haug, 1996, for a review). Note, however, that, when facing multivariate systems, the possibility that various cointegrating relations exist needs to be allowed for, so, at first sight, single-equation regression approaches appear not to be useful. Single-equation tests can be extended to systems of equations (see Ahn and Choi, 1995), but this would still require imposing identifying restrictions to design cointegrating regressions, so the application of such methods in practice remains speculative.

In this paper we develop a procedure to infer the cointegrating rank and to design a set of regressions from which the cointegrating vectors in system frameworks can be estimated. Our analysis focuses on $I(1)$ systems (where, after differencing and possibly eliminating time-varying deterministic components, the vector of observables is covariance stationary with nonzero and bounded spectral density), but our method allows for simple extensions to higher order settings, although we do not pursue this here.

Given the abundant literature on cointegration, it is warranted that we highlight the extent of our contribution. First, our proposal requires neither the imposition of a priori identifying conditions nor the specification of a parametric model for the short run components. There are procedures in the literature which achieve a similar goal, like the principal components approach (see Stock and Watson, 1988; Harris, 1997; Snell, 1999), the nonparametric method of Bierens (1997), or the test of common stochastic trends of Nyblom and Harvey (2000). However, while the main focus of these proposals is to test for a particular cointegrating rank (denoted throughout as r), they do not provide a formal discussion of estimation of r (with the exception of Bierens, 1997), and their estimators of the cointegrating vectors are based on eigenvalues routines and orthogonality restrictions, which might be difficult to interpret. We offer a formal discussion of the properties of our estimator of r, and, in addition, this estimator is based on simple techniques (like unit root testing) which belong to the standard time series toolkit. Second, once the cointegrating rank is determined, our method provides data-based just-identifying restrictions which lead to a Phillips' triangular form. In particular, our proposal identifies automatically the set of regressors from which a Phillips' triangular form can be straightforwardly estimated without imposing any a priori identification conditions. Thus, in practice, we provide a method which makes the application of Phillips' approach feasible, hence offering a valid alternative to Johansen's methodology. We believe there are relevant contexts where our proposal might indeed enjoy advantages over Johansen's. In particular, avoiding parametric assumptions on the short-run dynamics makes the method more robust to misspecification. It also seems preferable in high-dimensional models, where parametric prescriptions would lead to estimating a very large number of parameters, thus possibly inducing small sample problems. Additionally, Gonzalo and Lee (1998) showed that residual-based cointegration tests are more robust than Johansen's likelihood ratio (LR) type of tests to empirically relevant departures from the model, such as autoregressive processes with roots (marginally) larger than unity or stochastic roots, mistaken order of integration of the system ($I(2)$ taken as $I(1)$ with drift), wrong choice of deterministic components or fractional processes. We believe our proposal might have advantages in these cases and provide some evidence based on Monte Carlo simulations. Finally, we shed light on the delicate issue of choosing appropriately the regressors in cointegrating regressions. Here, our results appear to be useful even in uni-equation (including bivariate) settings, where residual-based cointegration testing is routinely applied by practitioners, but where a wrong design of the possible cointegrating regression (due to cointegrated regressors) might lead to erroneous conclusions.

The rest of the paper is organized as follows. In Section 2 we introduce some preliminary concepts and a result on which our methodology is based. In Section 3 we present a method to select common trends which, as will be seen below, is an essential component of our estimator of r. This estimator is introduced and its properties are discussed in Section 4. Next, in Section 5, we compare the finite sample performance of our procedure with that of Johansen's trace test (see, e.g. Johansen, 1995). In Section 6 we discuss an empirical analysis of the term structure of US interest rates and, finally, in Section 7, we conclude. Proofs of theorems are relegated to the Appendix.

2. Preliminary concepts and results

We first introduce some definitions. We say that a scalar or vector process ξ_t is integrated of order zero ($\xi_t \sim I(0)$) if $\xi_t - E(\xi_t)$ is covariance stationary with nonzero and bounded spectral density at all frequencies. Then, a scalar or vector ζ_t is integrated of order one ($\zeta_t \sim I(1)$), if $\Delta\zeta_t$ is $I(0)$, where $\Delta = 1 - L$, L being the lag operator. Note that if a vector ζ_t is $I(1)$, our definition (which is almost identical to that of Johansen, 1995) implies that at least one of the individual components of ζ_t is $I(1)$, but, in general, an $I(1)$ vector is allowed to have individual components with distinct integration orders.

Next, we define cointegration for $I(1)$ processes. Given a $p \times 1$ process $z_t \sim I(1)$, z_t is cointegrated if there exists a $p \times 1$ vector $\gamma \neq 0$ such that $\gamma' z_t - E(\gamma' z_t)$ (prime denoting transposition) can be made covariance stationary by a suitable choice of initial values. Hereafter, a process which can be made covariance stationary (or $I(0)$) by a suitable choice of initial values will just be denoted as stationary (or $I(0)$). Again, this definition is almost identical to that of Johansen (1995), and it is significantly more general than the standard notion of Engle and Granger (1987), where all observables are required to have identical integration orders. Note that, according to our definition, some of the cointegrating vectors might be trivial, just indicating that a particular observable is stationary (possibly after eliminating time-varying deterministic components). Also, note that $\gamma' z_t$ need not be $I(0)$ (e.g. if $\gamma' z_t - E(\gamma' z_t)$ is noninvertible). As usual, the cointegrating rank (r) among the elements of z_t is the number of linearly independent cointegrating vectors, and the space generated by these vectors (whose dimension is r) will be denoted as cointegrating space.

A very general model which generates a possibly $I(1)$ and cointegrated $p \times 1$ vector of observables z_t is

$$\Upsilon \Delta (z_t - \mu_t) = u_t, \qquad (1)$$

where Υ is a $p \times p$ nonsingular matrix, μ_t is a deterministic component and u_t is a zero-mean $p \times 1$ covariance stationary process which satisfies one (and only one) of the following conditions:

(i) u_t is $I(0)$ with nonsingular spectral density matrix at all frequencies;

(ii) some components of u_t form a subvector which is $I(0)$ with nonsingular spectral density matrix at all frequencies, the rest of the components forming another subvector which is the first difference of a zero-mean stationary process with bounded spectral density matrix at all frequencies;

(iii) u_t is the first difference of a zero-mean stationary process with bounded spectral density matrix at all frequencies.

We also set $E(z_0) = \mu_0$, so (1) immediately implies that $E(z_t) = \mu_t$, $t \geq 1$. In (1), the integration and cointegration properties of z_t depend both on Υ and u_t. For example, under (i) or (ii),