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Empirical likelihood for regression discontinuity design☆

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ABSTRACT

This paper proposes empirical likelihood based inference methods for causal effects identified from regression discontinuity designs. We consider both the sharp and fuzzy regression discontinuity designs and treat the regression functions as nonparametric. The proposed inference procedures do not require asymptotic variance estimation and the confidence sets have natural shapes, unlike the conventional Wald-type method. These features are illustrated by simulations and an empirical example which evaluates the effect of class size on pupils' scholastic achievements. Furthermore, for the sharp regression discontinuity design, we show that the empirical likelihood statistic admits a higher-order refinement, so-called the Bartlett correction. Bandwidth selection methods are also discussed.

1. Introduction

Since the seminal work of Thistlethwaite and Campbell (1960), regression discontinuity design (RDD) analysis has been a fundamental tool to investigate causal effects of treatment assignments on outcomes of interest. There are numerous methodological developments and empirical applications of RDD analysis particularly in the fields of economics, psychology, and statistics (see e.g. Trochim, 2001; Imbens and Lemieux, 2008, for surveys). The main purpose of this paper is to propose a new inference approach to RDD analysis based on empirical likelihood.[1]

In the literature of RDD analysis, there are at least two important issues that have attracted substantial attention from researchers. First, although RDD analysis were initially discussed in the context of regression analysis, recent research has focused on deeper understanding of the estimated parameters of interest based on the theory of causal effects (see e.g. Rubin, 1974; Holland, 1986; Angrist et al., 1996). In causal analysis, RDDs are split into two categories, the sharp and fuzzy RDDs. This categorization is based on how the treatment assignments are determined by a covariate (called the forcing variable). For the sharp design, the treatment is completely determined by the forcing variable on the either side of a cutoff value and we can identify and estimate the average causal effect of the treatment at the cutoff value. For the fuzzy design, the treatment is partly determined by the forcing variable and the treatment assignment probability jumps at the cutoff value. In this case, we can identify and estimate the average causal effect of the treatment for the compliers (see Hahn et al., 2001, and Section 2.1 below). The present paper adopts this framework and focuses on inferences for the average causal effects identified in the sharp and fuzzy RDDs.

The second issue that has attracted researchers' attention is the importance of nonparametric methods in RDD analysis (e.g. Sacks and Ylvisaker, 1978; Knafl et al., 1985). Since RDD analysis is concerned with the causal effects locally at some cutoff value of the

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1 See Owen (2001) for a review on empirical likelihood.

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forcing variable, it is natural to allow flexible functional forms for regression and treatment assignment probability functions. Hahn et al. (2001) and Porter (2003) proposed nonparametric estimators for average causal effects in the sharp and fuzzy RDDs based on local polynomial fitting (Fan and Gijbels, 1996). Their nonparametric estimators possess reasonable convergence rates and are asymptotically normal under certain regularity conditions. However, the asymptotic variances of these estimators, which are required to construct Wald-type confidence sets, are rather complicated due to discontinuities in the conditional mean, variance, and covariance functions. Typically, in order to estimate the asymptotic variances, we need additional nonparametric regressions to estimate the left and right limits of the conditional variances and covariances, and we also need nonparametric density estimation for the forcing variable.

In this paper we construct empirical likelihood-based confidence sets for causal effects identified from the sharp and fuzzy RDDs. Our empirical likelihood approach allows for nonparametric regression functions but does not require complicated asymptotic variance estimation. The proposed confidence sets have natural shapes, unlike the conventional Wald-type method. These features are illustrated by simulations and an empirical example which evaluates the effect of class size on pupils' scholastic achievements. We study the first- and second-order asymptotic properties of the empirical likelihood-based inference. We show that the empirical likelihood ratios for the causal effects in the sharp and fuzzy RDDs are asymptotically chi-square distributed. Therefore, similar to the existing papers such as Chen and Qin (2000) and Fan et al. (2001), we can still observe an analog of the Wilks phenomenon in this nonparametric RDD setup. Furthermore, for the sharp RDD setup, we study second-order asymptotic properties of the empirical likelihood ratio statistic and show that the empirical likelihood confidence set admits a second-order refinement, so-called the Bartlett correction. Bartlett correctability can be considered as an additional rationale of our empirical likelihood approach.[2]

The paper is organized as follows. In Section 2 we present the basic setup and construct the empirical likelihood function for the causal effects. Section 3 studies first-order asymptotic properties of the empirical likelihood ratios and confidence sets. Section 4 analyzes second-order properties of the empirical likelihood statistic for the sharp RDD setup. Section 5 discusses bandwidth selection methods. The proposed methods are examined in Section 6 through Monte Carlo simulations and an empirical example which evaluates the effect of class size on pupils' scholastic achievements investigated in Angrist and Lavy (1999). Section 7 concludes. Appendix contains the proofs, lemmas, and derivations for the main theorems.

2. Setup and methodology

2.1. Regression discontinuity design

We first introduce our basic setup. Let $Y_i(1)$ and $Y_i(0)$ be potential outcomes of unit i with and without exposure to a treatment, respectively. Let $W_i \in \{0, 1\}$ be an indicator variable for the treatment. We set $W_i = 1$ if unit i is exposed to the treatment and set $W_i = 0$ otherwise. The observed outcome is $Y_i = (1 - W_i) Y_i(0) + W_i Y_i(1)$ and we cannot observe $Y_i(0)$ and $Y_i(1)$ simultaneously. Our purpose is to make inference on the causal effect of the treatment, or more specifically, probabilistic aspects of the difference of potential outcomes $Y_i(1) - Y_i(0)$. RDD analysis focuses on the case where the treatment assignment W_i is completely or partly determined by some observable covariate X_i, called the forcing variable. For example, to study the effect of class size on pupils' achievements, it is reasonable to consider the following setup: the unit i is school, Y_i is an average exam score, W_i is an indicator variable for the class size ($W_i = 0$ for one class and $W_i = 1$ for two classes), and X_i is the number of enrollments.

Depending on the assignment rule for W_i based on X_i, we have two cases, called the sharp and fuzzy RDDs. In the sharp RDD, the treatment is deterministically assigned based on the value of X_i, i.e.

$$W_i = \mathbb{I}\{X_i \geq c\},$$

where $\mathbb{I}\{\cdot\}$ is the indicator function and c is a known cutoff point. A parameter of interest in this case is the average causal effect at the discontinuity point c,

$$\theta_s = \mathrm{E}[Y_i(1) - Y_i(0) | X_i = c].$$

Since the difference of potential outcomes $Y_i(1) - Y_i(0)$ is unobservable, we need a tractable representation of θ_s in terms of quantities that can be estimated by data. If the conditional mean functions $\mathrm{E}[Y_i(1) | X_i = x]$ and $\mathrm{E}[Y_i(0) | X_i = x]$ are continuous at $x = c$, then the average causal effect θ_s can be identified as a contrast of the right and left limits of the conditional mean $\mathrm{E}[Y_i | X_i = x]$ at $x = c$,

$$\theta_s = \lim_{x \downarrow c} \mathrm{E}[Y_i | X_i = x] - \lim_{x \uparrow c} \mathrm{E}[Y_i | X_i = x]. \tag{1}$$

In contrast to sharp RDD analysis, fuzzy RDD analysis focuses on the case where the forcing variable X_i is not informative enough to determine the treatment W_i but can affect the treatment probability. In particular, the fuzzy RDD assumes that the conditional treatment probability of W_i jumps at $X_i = c$,

$$\lim_{x \downarrow c} \Pr\{W_i = 1 | X_i = x\} \neq \lim_{x \uparrow c} \Pr\{W_i = 1 | X_i = x\}.$$

To define a reasonable parameter of interest for the fuzzy case, let $W_i(x)$ be a potential treatment for unit i when the cutoff level for the treatment was set at x, and assume that $W_i(x)$ is non-increasing in x at $x = c$. Using the terminology of Angrist et al. (1996), unit i is called a complier if her cutoff level is X_i, i.e.[3]

$$\lim_{x \downarrow X_i} W_i(x) = 0, \qquad \lim_{x \uparrow X_i} W_i(x) = 1.$$

A parameter of interest in the fuzzy RDD, suggested by Hahn et al. (2001), is the average causal effect for compliers at $X_i = c$,

$$\theta_f = \mathrm{E}[Y_i(1) - Y_i(0) | i \text{ is complier}, X_i = c].$$

Hahn et al. (2001) showed that under mild conditions the parameter θ_f can be identified by the ratio of the jump in the conditional mean of Y_i at $X_i = c$ to the jump in the conditional treatment probability at $X_i = c$, i.e.

$$\theta_f = \frac{\lim_{x \downarrow c} \mathrm{E}[Y_i | X_i = x] - \lim_{x \uparrow c} \mathrm{E}[Y_i | X_i = x]}{\lim_{x \downarrow c} \Pr\{W_i = 1 | X_i = x\} - \lim_{x \uparrow c} \Pr\{W_i = 1 | X_i = x\}}. \tag{2}$$

If additional covariates Z_i are available, the same identification arguments for θ_s and θ_f go through by slightly modifying the assumptions and adding conditioning variables $Z_i = z$ to the conditional means and probabilities above. This paper focuses on how to make inference for these average causal effect parameters θ_s and θ_f in the sharp and fuzzy RDDs.

To estimate the parameters θ_s and θ_f, it is common to apply some nonparametric regression techniques (e.g. Hahn et al.,

[2] Baggerly (1998) showed that for testing the mean parameter, only empirical likelihood is Bartlett correctable in the power divergence family.

[3] If $\lim_{x \downarrow X_i} W_i(x) = 0$ and $\lim_{x \uparrow X_i} W_i(x) = 0$, then unit i is called a nevertaker. If $\lim_{x \downarrow X_i} W_i(x) = 1$ and $\lim_{x \uparrow X_i} W_i(x) = 1$, then unit i is called an alwaystaker.