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Robust score and portmanteau tests of volatility spillover

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ABSTRACT

This paper presents a variety of tests of volatility spillover that are robust to heavy tails generated by large errors or GARCH-type feedback. The tests are couched in a general conditional heteroskedasticity framework with idiosyncratic shocks that are only required to have a finite variance if they are independent. We negligibly trim test equations, or components of the equations, and construct heavy tail robust score and portmanteau statistics. Trimming is either simple based on an indicator function, or smoothed. In particular, we develop the tail-trimmed sample correlation coefficient for robust inference, and prove that its Gaussian limit under the null hypothesis of no spillover has the same standardization irrespective of tail thickness. Further, if spillover occurs within a specified horizon, our test statistics obtain power of one asymptotically. We discuss the choice of trimming portion, including a smoothed *p*-value over a window of extreme observations. A Monte Carlo study shows our tests provide significant improvements over extant GARCH-based tests of spillover, and we apply the tests to financial returns data. Finally, based on ideas in Patton (2011) we construct a heavy tail robust forecast improvement statistic, which allows us to demonstrate that our spillover test can be used as a model specification pre-test to improve volatility forecasting.

1. Introduction

A rich literature has emerged on testing for financial market associations, spillover and contagion, and price/volume relationships during volatile periods (King et al., 1994; Karolyi and Stulz, 1996; Brooks, 1998; Comte and Lieberman, 2000; Hong, 2001; Forbes and Rogibon, 2002; Caporale et al., 2005, 2006). Similarly, evidence for heavy tails across disciplines is substantial, with a large array of studies showing heavy tails and random volatility effects in financial returns. See Campbell and Hentschel (1992), Engle and Ng (1993), Embrechts et al. (1999); Longin and Solnik (2001), Finkenstadt and Rootzen (2003), and Poon et al. (2003).

The ability to detect volatility spillovers among asset prices has myriad uses in macroeconomics and finance. For policy makers, knowledge of spillovers may inform policy design (King et al., 1994; Forbes and Rogibon, 2002). For investors, knowledge of spillovers may lead to improved volatility forecasts, which can be embedded inside asset pricing models. Similarly, spillovers might capture information transmission, as per Engle et al. (1990), or the spillover effects can be used to design conditional hedge ratios (Chang et al., 2011).

Non-correlation based methods have evolved in response to mounting evidence for heavy tails and heteroskedasticity in financial markets, including distribution free correlation-integral tests (Brock et al., 1996; de Lima, 1996; Brooks, 1998), exact small sample tests based on sharp bounds (Dufour et al., 2006), copula-based tests (Schmidt and Stadtmüller, 2006), and tail dependence tests (Davis and Mikosch, 2009; Hill, 2009; Longin and Solnik, 2001; Poon et al., 2003; Malevergne and Sornette, 2004).

1.1. Proposed methods

In this paper, rather than look for new dependence measures, we exploit robust methods that allow for the use of existing representations of so-called volatility *spillover* or *contagion*[1] where

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[1] There is some consensus in the applied literature on the use of the terms "spillover" versus "contagion" in financial markets: spillover concerns "usual" market linkages and contagion suggests "unanticipated transmission of shocks" (e.g. Beirne et al., 2008, p. 4). We simply use the term "spillover" for convenience and in view of past usage in the volatility literature (e.g. Cheung and Ng, 1996; Hong, 2001). Since we allow for very heavy tails in the errors, our contributions arguably also apply to the contagion literature since such noise renders anticipating linkages exceptionally difficult.

idiosyncratic shocks may be heavy tailed. We use a general model of conditional heteroskedasticity, and deliver test statistics with standard limit distributions under mild regularity conditions.

Let $\{y_{1,t}, y_{2,t}\}$ be a joint process of interest with conditionally heteroskedastic coordinates:

$$y_{i,t} = h_{i,t}(\theta_i)\epsilon_{i,t}(\theta_i) \quad \text{where } \theta_i \in \mathbb{R}^q \; q \geq 1. \tag{1}$$

We assume there exists a unique point θ_i^0 in the interior of a compact subset $\Theta \subset \mathbb{R}^q$ such that $\{y_{i,t}, h_{i,t}(\theta_i^0)\}$ is stationary and ergodic, and $E[\epsilon_{i,t}(\theta_i^0)] = 0$ and $E[\epsilon_{i,t}^2(\theta_i^0)] = 1$. Now drop θ_i^0 and write $h_{i,t} = h_{i,t}(\theta_i^0)$ and $\epsilon_{i,t} = \epsilon_{i,t}(\theta_i^0)$. Volatility $h_{i,t}(\theta_i)$ is measurable with respect to $\{y_{i,t-1}, y_{i,t-2}, \ldots\}$, continuous and differentiable on Θ, and bounded $\inf_{\theta\in\Theta}\{h_{i,t}(\theta_i)\} > 0$ *a.s.* An example of (1) is nonlinear GARCH(1, 1) $h_{i,t}^2 = g(y_{i,t}, h_{i,t-1}^2, \theta_i^0)$ where $g(\cdot, \cdot, \theta_i)$ is continuous (see Francq and Zakoïan, 2010). We restrict attention to models where random volatility $h_{i,t}^2$ satisfies

$$E\left(\sup_{\theta_i\in\mathcal{N}_0}\left\|\frac{\partial}{\partial\theta_i}\ln h_{i,t}^2(\theta_i)\right\|\right)^2 < \infty$$

$$\text{on some compact subset } \mathcal{N}_0 \subseteq \Theta \text{ containing } \theta_i^0. \tag{2}$$

Condition (2) simplifies technical arguments, but it can be relaxed at the expense of lengthier proofs. Since we want to allow for heavy tailed $\epsilon_{i,t}$, notice (2) in general implies $h_{i,t}^2$ is stochastic, since otherwise for many models $(\partial/\partial\theta)\ln h_{i,t}^2(\theta)|_{\theta=\theta_i^0}$ is square integrable only if $E[\epsilon_{i,t}^4] < \infty$ (Francq and Zakoïan, 2004, 2010). This allows us to avoid boundary issues for estimating θ_i^0 (for the GARCH case, see Andrews (2001)). In the standard GARCH model $h_{i,t}^2 = \omega_i^0 + \alpha_i^0 y_{i,t-1}^2 + \beta_i^0 h_{i,t-1}^2$ with $\omega_i^0 > 0$, for example, if $\alpha_i^0 + \beta_i^0 > 0$ then (2) holds (cf. Francq and Zakoïan, 2004), while in general (2) covers linear, Quadratic, GJR, Smooth Transition, Threshold, and Asymmetric GARCH, to name a few. See Engle and Ng (1993); Glosten et al. (1993); Sentana (1995) and Francq and Zakoïan (2010). It is only a matter of notation to allow even greater model generality, including nonlinear ARMA–GARCH and other volatility models (e.g. Meddahi and Renault, 2004).

Cheung and Ng (1996) and Hong (2001) work with a linear GARCH model $h_{i,t}^2 = \omega_i^0 + \alpha_i^0 y_{i,t-1}^2 + \beta_i^0 h_{i,t-1}^2$ and argue volatility spillover reduces to testing whether $y_{1,t}^2/h_{1,t}^2 - 1$ and $y_{2,t-h}^2/h_{2,t-h}^2 - 1$ are correlated for some lag $h \geq 1$, where $\epsilon_{i,t}$ is assumed to be serially independent. Hong (2001) proposes a standardized portmanteau statistic to test for spillover at asymptotically infinitely many lags, and requires $E[\epsilon_{i,t}^8] < \infty$, although $y_{i,t}$ may be IGARCH or mildly explosive GARCH, as long as $y_{i,t}$ is stationary.

The assumption of thin tails is not unique to these works since volatility spillover and contagion methods are typically designed under substantial moment conditions. Forbes and Rogibon (2002) implicitly require VAR errors to have a fourth moment; Caporale et al. (2005) exploit QML estimates for a GARCH model and therefore need at least $E[\epsilon_{i,t}^4] < \infty$, cf. Francq and Zakoïan (2004). Despite the fixation on thin-tail assumptions, in applications there appears to be little in the way of robustness checks, or pre-tests to verify the required moment conditions. See especially de Lima (1997) and Hill and Aguilar (2013). Dungey et al. (2005), for example, study an array of sampling properties of tests of contagion and spillover, but do not treat heavy tails. In Section 6, however, we show a variety of asset return series have conditionally heteroskedastic components with errors $\epsilon_{i,t}$ that may have an unbounded fourth moment.

Our approach is similar to Cheung and Ng (1996) and Hong (2001). We construct centered squared errors from the volatility function $h_{i,t}(\theta_i)$,

$$\mathcal{E}_{i,t}(\theta_i) \equiv \frac{y_{i,t}^2}{h_{i,t}^2(\theta_i)} - 1 = \epsilon_{i,t}^2(\theta_i) - 1 \quad \text{and} \quad \mathcal{E}_{i,t} = \mathcal{E}_{i,t}(\theta_i^0)$$

and build test equations over H lags:

$$m_t(\theta) = \left[m_{h,t}(\theta)\right]_{h=1}^H = \left[\mathcal{E}_{1,t}(\theta_1) \times \mathcal{E}_{2,t-h}(\theta_2)\right]_{h=1}^H, \quad H \geq 1,$$
$$\text{and} \quad m_t = m_t(\theta^0).$$

Under the null of no spillover we have $E[m_{h,t}] = E[(\epsilon_{1,t}^2 - 1)(\epsilon_{2,t-h}^2 - 1)] = 0$. The conventional assumption $E[m_{h,t}^2] < \infty$ requires $E[\epsilon_{i,t}^4] < \infty$ if $\epsilon_{1,t}$ and $\epsilon_{2,t}$ are mutually independent, while $E[\epsilon_{i,t}^8] < \infty$ is imposed to ensure consistency of estimated higher moments $E[m_{h,t}^2]$ given the presence of a plug-in for θ^0.

We conquer the problem of possibly heavy tailed non-iid shocks $\epsilon_{i,t}$ by transforming $m_t(\theta)$, $\mathcal{E}_{i,t}(\theta_i)$ or $\epsilon_{i,t}(\theta_i)$. First, since $m_{h,t}(\theta)$ is asymmetrically distributed about zero in general, we need an asymmetric transform to ensure both identification of the hypotheses and a standard distribution limit (cf. Hill, 2012, 2014a; Hill and Aguilar, 2013). We therefore focus on tail-trimming $m_{h,t}(\theta)I(-l \leq m_{h,t}(\theta) \leq u)$ for a robust score test, where $I(\cdot)$ is the indicator function, l and u are positive thresholds, and $l, u \to \infty$ as the sample size $T \to \infty$. In general, this does not allow a portmanteau statistic even if $\epsilon_{i,t}$ are iid, and may still lead to small sample bias that arises from trimming. Further, if l and u are fixed asymptotically then, in general, asymptotic bias in the test statistic prevents a score statistic from detecting spillover. By negligible trimming, however, we can obtain both an asymptotic chi-squared distribution under the null and correctly identify spillover. In principle other transformations can be used, including those discussed below for our portmanteau tests, but the need for asymmetry and negligibility makes tail-trimming an appealing and practical choice.

Our second and third approaches transform $\mathcal{E}_{i,t}(\theta_i)$ and $\epsilon_{i,t}(\theta_i)$, respectively, leading to robust score *and* portmanteau statistics. Small sample bias is eradicated by recentering the transformed variables. We use a class of bounded transformations $\psi : \mathbb{R} \times [0, \infty) \to \mathbb{R}$, $|\psi(u, c)| \leq c$, including the so-called *redescending* functions, which generate decreasing or vanishing values far from a threshold c, e.g. $\psi(u, c) = 0$ if $|u| > c$. We say ψ is *symmetric* if $\psi(-u, c) = -\psi(u, c)$, and we say the transformation ψ or threshold c is *negligible* when $\lim_{c\to\infty}\psi(u, c) = u$ such that there is no transformation asymptotically. We assume symmetry and negligibility throughout.

Redescenders are popularly used in the outlier robust estimation literature where an extreme value is considered aberrant. See Andrews et al. (1972), Hampel et al. (1986), and Jureckova and Sen (1996) for classic treatments, and for use in M-estimation see Kent and Tyler (1991); Shevlyakov et al. (2008) and Hill (2013b, 2014a). Examples of popularly used symmetric transforms ψ are *simple trimming* $uI(|u| \leq c)$, *Tukey's bisquare* $u(1 - (u/c)^2)^2 I(|u| \leq c)$, *exponential* $u\exp\{-|u|/c\}I(|u| \leq c)$, and *truncation sign*$\{u\}\min\{|u|, c\}$. Notice only the first three are redescenders.

By recentering $\psi(\mathcal{E}_{i,t}(\theta_i), c) - E[\psi(\mathcal{E}_{i,t}(\theta_i), c)]$ or $\psi(\epsilon_{i,t}^2(\theta_i), c) - E[\psi(\epsilon_{i,t}^2(\theta_i), c)]$ we can always use a symmetric transformation which is intrinsically easier to implement. Moreover, if $\epsilon_{1,t}$ and $\epsilon_{2,t-h}$ are independent under the null then ψ does not need to be redescending, nor even negligible in the sense that c may be bounded, since the null hypothesis is identified with any bounded ψ or any c. This allows for great generality in terms of possible Q-statistic constructions, and as a bonus ensures infinitesimal robustness when c is fixed. In order to conserve space, we do not formally treat data contamination in this paper. See Section 1.2 for further discussion. In practice, however, unless we know the error distribution for a simulation based bias correction or to model the bias (e.g. Ronchetti and Trojani, 2001; Mancini et al., 2005), only negligibility $c \to \infty$ as $T \to \infty$ ensures we capture spillover $E[m_{h,t}] \neq 0$ when it occurs. For example, we can always use simple trimming $uI(|u| \leq c)$ or the exponential $u\exp\{-|u|/c\}I(|u| \leq c)$ with