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Estimation of fixed effects panel regression models with separable and nonseparable space–time filters[☆]

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ABSTRACT

This paper considers a quasi-maximum likelihood estimation for a linear panel data model with time and individual fixed effects, where the disturbances have dynamic and spatial correlations which might be spatially stable or unstable. We first consider both separable and nonseparable space–time filters for the stable model. The separable space–time filter is subject to a parametric restriction which results in relative computational simplicity. In contrast to the spatial econometrics literature, we expose economic restrictions imposed by the separable space–time filter model and explore computational tractability of the nonseparable filter model. Throughout this paper, the effect of initial observations is taken into account, which results in an exact likelihood function for estimation. This is important when the span of time periods is short. We then investigate spatial unstable cases, where we propose to apply a "spatial differencing" to all variables in the regression equation as a data transformation, which may eliminate unstable or explosive spatial components in order to achieve a robust estimator. For estimates of the parameters in both the regression part and the disturbance process, they are $\sqrt{nT}$-consistent and asymptotically centered normal regardless of whether T is large or not and whether the process is stable or not.

1. Introduction

Panel regression models can be augmented with serial correlation or spatial dependence so as to control for time and spatial dependence in addition to heterogeneity in panels. These spatio-temporal interactions can be specified in dependent variables as in Su and Yang (2007), Yu et al. (2008), Elhorst (2010) among others. They can also be specified in the error components such as in Elhorst (2004), Baltagi et al. (2007), Parent and LeSage (2011, 2012) and Lee and Yu (2012). Empirical applications of these spatio-temporal dependences can be found in habit formation (Korniotis, 2010), growth convergence of countries and regions (Ertur and Koch, 2007; Mohl and Hagen, 2010), regional markets (Keller and Shiue, 2007), labor economics (Lottmann, 2012), public economics (Revelli, 2001; Franzese, 2007) and other areas of study. The current paper focuses on a panel regression model with serially and spatially correlated disturbances. In the spatial literature, the serial and spatial correlations in disturbances have often been separated. Baltagi et al. (2007) consider testing serial and spatial correlations in a model where these correlations are separable in disturbances. Parent and LeSage (2011, 2012) term it a space–time filter and consider Bayesian estimation. However, this separability imposes restrictions on spatial and time dynamics. It also rules out the possibility of spatial cointegration (Yu et al., 2012) as well as cases in which serial and spatial correlations may operate only through time, which is known as diffusion. In terms of forecasting, using spatial correlations improves forecasting performance, as noted in Giacomini and Granger (2004) and Longhi and Nijkamp (2007). Indeed, Giacomini and Granger (2004) conclude that ignoring spatial

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correlation leads to highly inaccurate forecasts in their application. The separable model does not involve any spatial effect in the best linear unbiased predictor formula, but it is present under the nonseparable model. From the estimation point of view, ignoring the mixing feature of space–time diffusion might result in estimating a misspecified dynamic process. The estimates of parameters in the misspecified disturbances may be subject to estimation bias and lead to inaccurate statistical inference when a mixing of time and space features is present in the data generating process (DGP).

The current paper investigates asymptotic properties of the quasi-maximum likelihood estimator (QMLE) for a panel regression model with dynamic and spatial correlations, where the separable space–time filter is a special case. The estimation of such a general space–time dynamic process has been argued (e.g., in Parent and LeSage (2011, 2012)) as unwise due to additional complexity in computation relative to the separable space–time filter for Bayesian estimation. Thus, we pay special attention to the computational issue of the nonseparable space–time model with the QML approach. In the estimation, we also consider the treatment of initial period observations, which is important when the number of time periods is small. This results in the formulation of an exact likelihood function. Indeed, the computational issue is raised due to the initial period observation. If the initial period observation were treated as exogenously given, the QML estimation would be conditional on the initial period observation. Due to the recursive nature of a dynamic process, the evaluation of the conditional likelihood would be computationally simple (Yu et al., 2008; Parent and LeSage, 2011, 2012). The conditional approach, however, is not proper in general for a short spatial panel.

For the disturbances in a panel regression model with a nonseparable space–time filter, they may have not only serial correlation and spatial correlation, but also possible space–time unstable or explosive features in certain circumstances, e.g., the study of market integration in Keller and Shiue (2007). For the general space–time dynamic process, some absolute summability conditions can be imposed if the process is stable in both space and time dimensions. However, a general nonseparable process might have spatial cointegration or explosive features when the eigenvalues in the DGP of the disturbances have some unit roots or even explosive ones (Holly et al., 2011; Yu et al., 2012). This calls for a treatment in the estimation because ordinary least square (OLS) or least square dummy variable (LSDV) estimates of the regression equation would not be consistent if there is non-stability in the disturbances.[1] Thus, in addition to the analysis of the general stable space–time dynamic process and that of a separable space–time filter, this paper also concerns possible space–time nonstationarity in the disturbances. We propose the use of time differencing and spatial differencing transformations to handle space–time nonstationarity in estimation. The spatial differencing transformation can be applied regardless of whether disturbances are spatially stable or not. With this data transformation, common inference can be performed and is robust without nonstandard asymptotic properties for estimates.

In addition to providing asymptotic properties of QML estimation and testing for a spatial panel with both separable and nonseparable space–time filters, the current paper has the following contributions to the literature. First, we consider the stochastic spatial time process in the disturbances. In Yu et al. (2008, 2012), the model considered has spatial and dynamic lags in the main regression. Its particular feature is the presence of fixed individual effects and/or exogenous variables that will generate time trend components in the final form of the dynamic model. With spatial nonstationarity, the time trend is a dominant feature for the asymptotic distribution of an estimator. The model in the current paper has a stochastic trend term but not a dominant time term. This difference could make asymptotic distribution of estimators different. The current paper provides some estimation methods which can overcome the spatial nonstationarity in terms of stochastic trend which may be present in a spatial nonstationarity process. To make the paper complete, we consider not only spatial nonstationarity but also stable and explosive cases. Thus, the current paper is different from our previous research on spatial cointegration for a spatial dynamic panel data (SDPD) model. Second, we consider the estimation based on a likelihood function that is exact in the sense that we do not need to approximate the initial observation. Lee and Yu (2010b, 2011) present the spatial difference transformation for the SDPD model but the estimation is conditional on the initial observation, and the consistency of those estimators requires that T tends to infinity. Furthermore, when T and n tend to infinity at the same rate, asymptotic biases in estimators exist. In the current paper, the initial condition is generated by the process itself and our estimation method is applicable to the fixed T situation. The estimators are shown to be consistent and asymptotically normal without T tending to infinity, and there are no asymptotic biases. Therefore, the analysis in the current paper can be applied to both fixed T and large T cases. Third, although there are some discussions on the use of the spatial difference transformation in Lee and Yu (2010b, 2011), rigorous justifications on asymptotic properties of estimators are not provided. The current paper provides rigorous analysis; in particular, we extend the inverse matrix of a generalized first difference matrix to the block matrix form (Hsiao et al., 2002). As a result, we can justify the uniform boundedness property of the involved matrix in order to relate the analysis to those established by Kelejian and Prucha (1998) for spatial econometric models.

The rest of the paper is organized as follows. Section 2 introduces the model and discusses the separable and nonseparable space–time filters. Economic implications of separable and nonseparable filters and distinctive characteristics of stability and space–time unstable features are presented. Section 3 studies the panel regression model with the separable space–time filter structure and its estimation,[2] and Section 4 investigates estimation of the general nonseparable time and space correlations in disturbances. Section 5 discusses the use of the spatial difference operator that can eliminate possible nonstationary features in the data and studies the resulting asymptotic properties of QMLE. Section 6 investigates the finite sample performance by Monte Carlo simulations of the QML estimates and classical tests for the constraint which characterizes the space–time filter. Section 7 concludes. Some algebraic derivations and proofs are collected in the Appendices.

2. A panel regression model with a general space–time filter

2.1. The model

Consider the model

$$Y_{nt} = X_{nt}\beta_0 + \mathbf{c}_{n0} + \alpha_{t0} l_n + U_{nt}, \quad t = 1, \ldots, T \tag{1}$$
$$U_{nt} = \lambda_0 W_n U_{nt} + \gamma_0 U_{n,t-1} + \rho_0 W_n U_{n,t-1} + V_{nt},$$

where $Y_{nt} = (y_{1t}, y_{2t}, \ldots, y_{nt})'$ and $V_{nt} = (v_{1t}, v_{2t}, \ldots, v_{nt})'$ are $n \times 1$ column vectors, and v_{it}'s are *i.i.d.* across i and t with zero mean and variance σ_0^2. W_n is an $n \times n$ nonstochastic spatial

[1] See Baltagi et al. (2008) for detailed discussions.

[2] While Parent and LeSage (2011) suggest the implementation of a Bayesian approach, we study the classical QML estimation and investigate asymptotic properties of the estimates.