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MODEL-BASED PRICING FOR FINANCIAL DERIVATIVES

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Assume that S_t is a stock price process and B_t is a bond price process with a constant continuously compounded risk-free interest rate, where both are defined on an appropriate probability space P. Let $y_t = \log(S_t/S_{t-1})$. y_t can be generally decomposed into a conditional mean plus a noise with volatility components, but the discounted S_t is not a martingale under P. Under a general framework, we obtain a risk-neutralized measure Q under which the discounted S_t is a martingale in this paper. Using this measure, we show how to derive the risk neutralized price for the derivatives. Special examples, such as NGARCH, EGARCH and GJR pricing models, are given. Simulation study reveals that these pricing models can capture the "volatility skew" of implied volatilities in the European option. A small application highlights the importance of our model-based pricing procedure.

1. Introduction. After the seminal work of Black and Scholes (1973) and Merton (1973), there has been explosive growth in the trading activities on derivatives in the worldwide financial markets. A fundamental question in finance is how we give a fair price for the derivative, whose payoff is on the evolution of an asset price upon which the derivative is written. Black and Scholes (1973) first fairly valued the option according to the principle of "the absence of arbitrage". Their valuation method relies on "efficient market hypothesis", under which there exists a risk-neutralized probability measure such that the discounted asset price is a martingale, and then a fair price of the derivative is the expected discounted value of its future payoff under this measure. Particularly, the risk-neutralized measure is not unique when the market is incomplete. For more discussions on the principle of "the absence of arbitrage", we refer to Harrison and Kreps (1979) and Harrison and Pliska (1982).

Although Black and Scholes's (1973) pricing model (hereafter, BS model) has achieved a great success in finance, it exhibits some systematic bias. The well-documented evidence is the so-called "volatility smile" in Rubinstein (1985) and Sheikh (1991), from which one may concern that the homoscedastic assumption on an asset return (that is, the asset return follows a geometric Brownian motion) is not reliable any more. This

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