Journal of Econometrics 187 (2015) 498–511

Contents lists available at ScienceDirect

Journal of Econometrics

journal homepage: www.elsevier.com/locate/jeconom

What is beneath the surface? Option pricing with multifrequency latent states

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ARTICLE INFO

Article history:
Available online 9 March 2015

JEL classification:
C51
G12
G13

Keywords:
Markov-switching multifractal
Particle filter
Regime-switching
Stochastic volatility
Jump-risk premium
Option pricing

ABSTRACT

We introduce a tractable class of multi-factor price processes with regime-switching stochastic volatility and jumps, which flexibly adapt to changing market conditions and permit fast option pricing. A small set of structural parameters, whose dimension is invariant to the number of factors, fully specifies the joint dynamics of the underlying asset and options implied volatility surface. We develop a novel particle filter for efficiently extracting the latent state from joint S&P 500 returns and options data. The model outperforms standard benchmarks in- and out-of-sample, and remains robust even in the wake of seemingly large discontinuities such as the recent financial crisis.

1. Introduction

Despite great progress in option pricing over the past thirty years, a central challenge still remains: to develop parsimonious price processes characterized by a small number of stable, well-identified parameters, yet with rich enough dynamics to match greatly varying shapes of the implied volatility (IV) surface at different dates. Latent factor models seem well suited to this challenge. Variations in the hidden state can drive variations in the underlying asset's conditional density and derivative prices, while a potentially small set of structural parameters remains fixed over time. In this paper, we develop a tractable class of regime-switching jump-diffusions, which are conditionally affine and permit fast option pricing. When the latent state exhibits multifrequency scaling, the resulting price process is tightly specified by a small number of fixed parameters and captures the rich dynamics of the IV surface both in- and out-of-sample.

The one-factor stochastic volatility models of Hull and White (1987) and Heston (1993) and the affine jump-diffusions of Bates (1996), Bakshi et al. (1997), and Duffie et al. (2000) have stimulated a large body of modern option pricing research. Nonetheless, one-factor models are now widely acknowledged to be too restrictive to capture important features of the IV surface.[1] Outside the option pricing literature, multiple volatility factors have proven to be useful for modeling the time-series of asset returns (Calvet et al., 1997; Engle and Lee, 1999; Calvet and Fisher, 2001, 2008a; Chernov et al., 2003), forecasting volatility (Calvet and Fisher, 2004; Calvet et al., 2006; Lux and Kaizoji, 2007; Lux, 2008; Corsi, 2009), and understanding equilibrium prices and returns (Calvet and Fisher, 2007; Adrian and Rosenberg, 2008; Campbell et al., 2012). These advances suggest that multi-factor volatility should play an increasingly important role in derivatives research.

A growing strand of the option literature incorporates multi-factor stochastic volatility within an *affine* asset-pricing framework.[2] While offering fast valuation, affine models entail important limitations for derivatives pricing (Li and Zhang, 2013). Non-affine multi-factor models offer the potential for greater flexibility, but are typically cumbersome to implement. For example,

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http://dx.doi.org/10.1016/j.jeconom.2015.02.034
0304-4076/

[1] See, for example, Bates (2000), Jones (2003), and Garcia et al. (2009), as well as the literature cited therein.

[2] See Andersen et al. (2012), Bates (2000, 2012), Christoffersen et al. (2009, 2010a), and Egloff et al. (2010).

the mathematical finance literature has studied Markov-switching jump-diffusions with diffusive volatility. Derivative prices are then usually determined by a system of partial differential equations that is challenging to solve numerically (Elliott et al., 2007a).[3] Because the empirical implementation of non-affine models is hindered by the cost of computing option prices, the analysis often only considers the time-series of the underlying asset, or simple volatility contracts such as variance swaps or the VIX, rather than a full options panel.[4]

In this paper, we develop a highly tractable class of Markov-switching jump-diffusions, which extend the Heston (1993) model to multiple stochastic volatility factors and jumps. Our approach builds on an earlier contribution by Dai and Singleton (2003), who consider an interest rate term-structure model with regime-switching in the long-run levels of the factors. We generalize their technology by allowing for jumps and apply the extended formulation to equity. In our general specification, the volatility of the underlying asset mean-reverts toward a stochastic central tendency driven by a Markov chain. We permit jumps in the underlying asset price that are tied to regime changes in the volatility state, as equilibrium valuation implies (Calvet and Fisher, 2008b). The resulting model of the stock price is *conditionally affine* and delivers fast option valuation.

To facilitate empirical implementation, we next introduce a tight specification of the conditionally affine price process. We assume that the latent state follows a Markov-switching Multifractal (MSM), as defined in Calvet and Fisher (2001, 2004). MSM is a parsimonious pure regime-switching model with volatility components of heterogeneous frequencies, which matches the fat tails, hyperbolic autocorrelation in volatility, and moment-scaling commonly observed in financial data. The present paper parsimoniously extends MSM by incorporating multifrequency diffusive volatility and leverage effects. We also propose a tight parametrization of jumps and risk premia, and call the resulting model *Skew MSM*. A striking property of Skew MSM is that it accommodates arbitrarily many volatility factors, jumps, and risk premia with only a small number of fixed structural parameters.

We develop new filtering and estimation techniques for option pricing. In a general state-space model, the joint density of returns and option prices can be used to impute latent states and efficiently estimate the structural parameters of the model. The bootstrap particle filter of Gordon et al. (1993) allows the empiricist to track a latent state by way of a recursive algorithm, which uses Monte Carlo draws (particles) to approximate the conditional state and data densities at each time-step.[5] In a model with a rich state space and large rare events, however, the bootstrap filter may not capture sudden moves to unlikely, but nonetheless important, parts of the state space. To address this issue we develop a variant of the bootstrap filter with stratified sampling, which ensures that all parts of the discrete MSM state space are represented. The particle filter permits efficient filtering and maximum likelihood estimation (MLE) of Skew MSM on the underlying asset and a panel of options.

This work is among the limited number of papers that estimate structural valuation models using both spot and option prices.[6] To the best of our knowledge, the present contribution is also the first to efficiently filter the latent state by using the joint distribution of the underlying asset returns and option IV surface. In comparison, among the leading prior applications of filtering to option pricing, Johannes et al. (2009) estimate jump-diffusions using stock return data only and incorporate short-maturity options into filters to generate diagnostics. Christoffersen et al. (2010b) filter latent states only from stock returns and use option data to evaluate goodness of fit and estimate parameters. The inference methods developed in this paper can be conveniently applied to Skew MSM because our model is parsimonious and permits fast option pricing.

Our empirical analysis begins by examining the performance of Skew MSM using only equity index returns. We conduct ML estimation on a long sample of the S&P 500 index, and find that the in-sample likelihood increases as frequency components are added, even though higher-dimensional specifications require no additional parameters. Skew MSM also fits the return data considerably better than standard jump-diffusions.

We next carry out joint estimation on both index returns and index options. Our in-sample options data is composed of almost ten years of monthly option surfaces, with a wide range of strikes and maturities. Skew MSM produces higher in-sample likelihood and lower root mean square pricing errors than benchmark affine jump-diffusions. In five years of out-of-sample data that include the recent financial crisis, Skew MSM again matches option prices more closely than standard benchmarks.

The remainder of the paper is organized as follows. Section 2 introduces a class of conditionally-affine regime-switching jump-diffusions and develops fast option pricing. Section 3 defines Skew MSM by parsimoniously specifying multi-factor volatility and risk premia for our conditionally affine process. Section 4 discusses the empirical methodology and develops a particle filter for stock and option data. Section 5 reports the empirical results. All proofs are in Appendix.

2. A tractable class of conditionally affine jump-diffusions

This section introduces a class of conditionally affine diffusions, which extends Heston's model to multiple volatility regimes and jumps.

2.1. A diffusion with regime-switch dependent jumps

We consider a frictionless financial market defined on the continuous time domain $\mathcal{T} = [0, \infty)$. The structure of uncertainty is specified by a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in\mathcal{T}}, \mathbb{P})$, where $\mathbb{P}$ denotes the physical measure. The economy is driven by a Markov chain $(M_t)_{t\in\mathcal{T}}$ taking values on a finite set $\mathcal{D} = \{m^1, \ldots, m^d\}$.

3 See also Chourdakis (2006) and Elliott et al. (2007b). Elliott et al. (2011) studies a restricted version of the equilibrium model developed in Calvet and Fisher (2008b). Empirically, Durham and Park (2013) circumvent the complexity of estimating regime-switching jump-diffusions on a full options panel by estimating their model on integrated volatility.

4 For example, Kaeck and Alexander (2012) estimate a variety of affine and non-affine models on S&P 500 index returns and the VIX term structure.

5 Applications of particle filtering to inference about hidden volatility states include Calvet et al. (2006), Johannes et al. (2009), Malik and Pitt (2011), and Christoffersen et al. (2010b).

6 Chernov and Ghysels (2000) use the Efficient Method of Moments. For the estimation, they only include the at-the-money call with the shortest time to maturity. Pan (2002) uses a tailored version of the Generalized Method of Moments (GMM) to estimate the Bates model using a time series of S&P 500 index returns and options (two per day). Eraker (2004) uses Markov Chain Monte Carlo methods to estimate the risk premia for jumps in returns and volatility, also using returns and options (around three per day). To our best knowledge, Broadie et al. (2007) were the first to consider the whole cross-section of option prices on the S&P 500 within an integrated approach. To reduce the computational burden, they fix some parameters using results from previous studies for the time series of returns and then use a least-square argument to fit option prices. Finally, Garcia et al. (2011) propose a GMM-based methodology for the Heston model that uses high-frequency data from the index returns and they apply it to the whole cross-section of option prices. However, it is not obvious how to extend their approach to include jumps.