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Consistent estimation of the fixed effects stochastic frontier model

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ABSTRACT

In this paper we consider a fixed-effects stochastic frontier model. That is, we have panel data, fixed individual (firm) effects, and the usual stochastic frontier analysis (SFA) composed error.

Maximum likelihood estimation (MLE) of this model has been considered by Greene (2005a,b). It is subject to the "incidental parameters problem", that is, to possible inconsistency due to the number of parameters growing with the number of firms. In the linear regression model with normal errors, it is known that the MLE of the regression coefficients is consistent, and the inconsistency due to the incidental parameters problem applies only to the error variance. Greene's simulations suggest that the same is true in the fixed effects SFA model.

In this paper we take a somewhat different approach. We consider MLE based only on the joint density of the deviations from means. In the linear regression model with normal errors, this estimator is the same as the full MLE for the regression coefficients, but it yields a consistent estimator of the error variance. For the SFA model, the MLE based on the deviations from means is not the same as the full MLE, and it has the advantage of not being subject to the incidental parameters problem.

The derivation of the joint density of the deviations from means is made possible by results in the statistical literature on the closed skew normal family of distributions. These results may be of independent interest to researchers in this area.

Simulations indicate that our within MLE estimator performs quite well in finite samples.

We also present an empirical example.

1. Introduction

In this paper we consider a fixed-effects stochastic frontier model of the form:

$$y_{it} = \alpha_i + X_{it}\beta + \varepsilon_{it}, \qquad \varepsilon_{it} = v_{it} - u_{it}, \quad u_{it} \geqslant 0. \tag{1}$$

Here $i = 1, \ldots, N$ indexes firms and $t = 1, \ldots, T$ indexes time periods. We have in mind a production frontier so that y is typically log output and X is a vector of functions of inputs. The v_{it} are iid $N\left(0, \sigma_v^2\right)$, the u_{it} are iid $N^+(0, \sigma_u^2)$ (i.e. half-normal), and X, v and u are mutually independent (so X can be treated as fixed). This is a fixed-effects model in the usual sense that no assumptions are made about the α_i, which we will refer to as the individual effects (or firm effects). They are regarded as fixed numbers that can be estimated as parameters, or eliminated by suitable transformation.

This model has been considered by Greene (2004, 2005a,b). A similar model was considered earlier by Polachek and Yoon (1996), and a different but closely related model is discussed in Kumbhakar and Wang (2005) and Wang and Ho (2010). The motivation for the model is that u_{it} represents technical inefficiency whereas α_i represents "heterogeneity" and presumably controls for time-invariant factors that affect the firm's output but that are not regarded as inefficiency (e.g. because they are not under the control of the firm). This is fundamentally different from earlier treatments, such as Pitt and Lee (1981) and Schmidt and Sickles (1984), in which inefficiency was time invariant and the only heterogeneity was the normal error v_{it}. For example, in Schmidt and Sickles there was no u_{it} and inefficiency was measured by the difference across firms in their individual effects α_i. Whether systematic time invariant differences in firm output more likely represent heterogeneity or inefficiency is an arguable point. However, in this paper we bypass these philosophical issues and

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concentrate instead on the technical question of how to estimate the model (1) consistently.

Greene (2005a) proposed the "true fixed effects" (TFE) estimator in which the α_i are estimated as parameters. More precisely, he maximizes the usual SFA likelihood function (based on the pdf of the ε_{it}) with respect to the parameters $\alpha_1, \ldots, \alpha_N \beta \sigma_v^2$ and σ_u^2. An unsolved question is whether this ML estimator is consistent, if asymptotics are understood to involve $N \to \infty$ (whether T is fixed or $T \to \infty$). The issue is the so-called "incidental parameters problem" which arises because the number of parameters depends on the sample size (there are N of the α_i).

There is no clear general answer to the question of for which models the fixed-effects MLE is consistent. For example, in the fixed-effects logit model, it is not consistent. For the fixed effects linear model with normal errors (i.e. the model above but without the u_{it}), which is arguably more similar to the present model, the situation is well understood. Here the MLE of β is consistent as $N \to \infty$, but the MLE of the error variance is inconsistent unless also $T \to \infty$. The asymptotic bias in the estimate of the error variance for finite T is easily corrected. Greene's simulations suggest (but obviously cannot prove) that the situation for the fixed-effects SFA model is similar. The MLE of β appears to be unbiased, but the MLE's of the error variances are biased. A difference between these results and the results for the linear model with normal errors is that in the present case there is no known simple correction for the error variance estimates. The error variances are important in the SFA context because they affect the extraction of estimated u from estimated ε (Jondrow et al., 1982).

In this paper we suggest an alternative to the TFE treatment of this model. Specifically, we propose a "within MLE" that maximizes the likelihood based on the joint density of the deviations from the individual means of the ε_{it}. That is, we remove the individual effects by the usual within transformation, and then apply MLE. In the linear model with normal errors, this would lead to the same estimate of β as the TFE treatment. Also, interestingly, it leads to a consistent estimator of the error variance for fixed T. In the SFA model, it does not lead to the same estimates of β or of the variance parameters as the TFE estimates. The point, of course, is that we have removed the individual effects by the within transformation, and the number of remaining parameters does not depend on N, so there is no incidental parameters problem. Subject to the usual types of regularity conditions on X, the within MLE should be consistent.

This is the same strategy as was followed in Wang and Ho (2010). The details are different because the models are different. In particular, the fact that in this paper u_{it} varies randomly over t (whereas in Wang and Ho the random portion of their u_{it} was time invariant) makes the distribution theory considerably more difficult.

The derivation of the joint density of the deviations from means of the ε_{it} is made possible by results in the statistical literature on the closed skew normal distribution. Our likelihood is more complicated than the usual SFA likelihood, but simple enough that MLE based on it is feasible. Our simulations indicate that the resulting estimates are quite reliable, and specifically that we do not encounter the bias in estimation of the variance parameters that Greene found for the TFE estimator.

The plan of the paper is as follows. In Section 2 we give a brief review of the linear regression model with normal errors, and we show that the within MLE of β is the same as the TFE estimate, but that the within estimator of the error variance is consistent as $N \to \infty$ with T fixed, unlike the TFE estimator. In Section 3 we provide a compendium of results on the closed skew normal family of distributions. In Section 4 we apply these to the fixed effects SFA model to construct the likelihood that our estimator will maximize. Section 5 gives the results of our simulations. In Section 6 we show the results of an empirical application. Section 7 concludes. There is also an appendix that contains some technical details and proofs.

2. Review of the fixed-effects linear model

In this section we provide a brief review of results for the linear regression model with fixed effects and normal errors. The model is the same as model (1) above, but without the one-sided error u. That is,

$$y_{it} = \alpha_i + X_{it}\beta + v_{it}, \tag{2}$$

where, as above, the X_{it} are treated as fixed and the v_{it} are iid normal. No assumptions are made about the individual effects α_i.

We will need some notation for means and deviations from means. For any variable z_{it}, we define the (individual) mean for firm i as $\bar{z}_i = \frac{1}{T}\sum_t z_{it}$, and we define the deviations from the individual means as $\tilde{z}_{it} = z_{it} - \bar{z}_i$. The transformation from z_{it} to $\tilde{z}_{it}$ is called the *within transformation*. Note that it annihilates time invariant variables; specifically, $\tilde{\alpha}_i = 0$.

The true fixed effects (TFE) estimator is least squares applied to (2), treating the parameters as $\alpha_1, \ldots, \alpha_N, \beta$. It is sometimes called OLS with dummy variables (OLSDV) because it is calculated as a regression of y on [X, dummy variables for the firms]. With normal errors, it is the MLE, and the MLE of σ_v^2 is $\hat{\sigma}_v^2 = \frac{1}{NT}\sum_i\sum_t(y_{it} - \hat{\alpha}_i - X_{it}\hat{\beta})^2$. The MLE of σ_v^2 is not consistent as $N \to \infty$ with T fixed, but a consistent estimate can be obtained by multiplying the MLE by $\frac{T}{T-1}$.

There are many estimators of β that are the same as the TFE estimator for this model, but which would not necessarily be the same as the TFE estimator for more complicated models like the fixed-effects SFA model. Here is a listing of some of them.

a. *Within estimator.* Perform the within transformation on Eq. (2) to obtain:

$$\tilde{y}_{it} = \tilde{X}_{it}\beta + \tilde{v}_{it}. \tag{3}$$

(Note that this transformation has removed α_i.) Then apply OLS to (3). Also, the TFE estimates of the α_i can then be recovered as

$$\hat{\alpha}_i = \bar{y}_i - \bar{X}_i\hat{\beta} \quad \text{where } \hat{\beta} \text{ is the within estimate.} \tag{4}$$

b. *IV* 1. Do instrumental variables on (2), where the instruments are $\tilde{X}_{it}$.

c. *IV* 2. Do instrumental variables on (3), where the instruments are X_{it}.

d. *Mundlak (1978)*. Regress y_{it} on $[X_{it}, \bar{X}_i]$. The estimated coefficients of X_{it} are the estimates of β.

e. *Chamberlain (1980)*. Regress y_{it} on $[X_{it}, X_{i1}, X_{i2}, \ldots, X_{iT}]$. The estimated coefficients of X_{it} are the estimates of β.

The point of this listing is to make clear that there are many estimators that equal the TFE estimator for the linear model with normal errors, but which would be different from the TFE estimator for the panel data SFA model. We will now define one other such estimator, which will be the one that we will extend to the panel data SFA model.

f. *Within MLE*. Maximize the likelihood given in Eq. (10), which is based on the joint density of the first $(T-1)$ deviations from individual means of the v_{it}.

To motivate this estimator, we first state some well-known results from the panel data literature. If the v_{it} in (2) are iid $N(0, \sigma_v^2)$, then (since $v_{it} = y_{it} - \alpha_i - X_{it}\beta$) the log likelihood for the model is

$$\ln L = \text{constant} - \frac{NT}{2}\ln\sigma_v^2 - \frac{1}{2\sigma_v^2}\sum_i\sum_t(y_{it} - \alpha_i - X_{it}\beta)^2. \tag{5}$$

Using the identity that, for any $z_1, \ldots, z_T$, $\sum_t z_t^2 = \sum_t(z_t - \bar{z})^2 + T\bar{z}^2$, we can factor this as:

$$\ln L = \text{constant} - \frac{NT}{2}\ln\sigma_v^2 - \frac{1}{2\sigma_v^2}SSE_W - \frac{T}{2\sigma_v^2}SSE_B \tag{6}$$