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New tools for understanding the local asymptotic power of panel unit root tests[☆]

Joakim Westerlund[a,b,*], Rolf Larsson[c]

[a] Lund University, Sweden

[b] Financial Econometrics Group, Centre for Research in Economics and Financial Econometrics, Deakin University, Australia

[c] Uppsala University, Sweden

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ABSTRACT

Motivated by the previously documented discrepancy between actual and predicted power, the present paper provides new tools for analyzing the local asymptotic power of panel unit root tests. These tools are appropriate in general when considering panel data with a dominant autoregressive root of the form $\rho_i = 1 + c_i N^{-\kappa} T^{-\tau}$, where $i = 1, \dots, N$ indexes the cross-sectional units, T is the number of time periods and c_i is a random local-to-unity parameter. A limit theory for the sample moments of such panel data is developed and is shown to involve infinite-order series expansions in the moments of c_i, in which existing theories can be seen as mere first-order approximations. The new theory is applied to study the asymptotic local power functions of some known test statistics for a unit root. These functions can be expressed in terms of the expansions in the moments of c_i, and include existing local power functions as special cases. Monte Carlo evidence is provided to suggest that the new results go a long way toward bridging the gap between actual and predicted power.

1. Motivation

Consider the problem of testing for a unit root in the panel data variable $\{y_{i,t}\}_{t=1}^{T}$, and assume for simplicity that the data generating process (DGP) is given by $y_{i,t} = \rho_i y_{i,t-1} + \varepsilon_{i,t}$, where $y_{i,0} = 0$ and $\varepsilon_{i,t} \sim N(0, 1)$. The analysis of the local power of various unit root test statistics when applied to such variables has attracted much attention in recent years (see Westerlund and Breitung, 2013, Section 2, for a review of this literature). The limit theory makes extensive use of the laws of large numbers and central limit theory, leading to local asymptotic power functions that are stated in terms of moments of various sample quantities. Almost all this theory assumes that $\rho_i = 1 + c_i N^{-\kappa} T^{-1}$, where $\kappa = 1/2$ or $\kappa = 1/4$, depending on whether the data contain incidental intercept and trend terms.[1] Without such terms, it has been found that in the above DGP with $\kappa = 1/2$ local power depends on the mean of c_i, but not on the variance, or any other moment for that matter (see, for example, Breitung, 2000; Moon and Perron, 2004; Moon et al., 2007). This means that one can just as well assume that $c_1 = \cdots = c_N = c$; there are no additional insights to be gained by allowing c_i to vary, at least not from a power point of view.

The fact that according to theory power should only depend on the mean of c_i is somewhat of an anomaly, because in Monte Carlo studies there is also a dependence on higher moments. Indeed, as Moon and Perron (2008, page 91) conclude from their simulation study, "despite our theoretical results, there is somewhat of a power loss against a heterogeneous alternative in finite samples" (see Moon et al., 2007, Section 7, for a similar finding). Let us illustrate this point using as an example the pooled ordinary least

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* Correspondence to: Department of Economics, Lund University, Box 7082, 220 07 Lund, Sweden. Tel.: +46 46 222 8997; fax: +46 46 222 4613.
E-mail address: joakim.westerlund@nek.lu.se (J. Westerlund).

[1] Moon and Phillips (1999) show that the maximum likelihood estimator of the local-to-unity parameter in near unit root panels with individual-specific trends is inconsistent. They call this phenomenon, which arises because of the presence of an infinite number of nuisance parameters, an "incidental trend problem", because it is analogous to the well-known incidental parameter problem in dynamic fixed-T panels.

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squares (OLS) t-statistic for a unit root, whose local asymptotic distribution (as $N, T \to \infty$) in the simple DGP considered here is given by

$$\frac{\mu_c^1}{\sqrt{2}} + N(0, 1), \tag{1}$$

where $\mu_c^1 = E(c_i)$ (see, for example, Moon et al., 2007, Section 3). If $\mu_c^1 < 0$ and the test is set up as left-sided, the asymptotic local power implied by (1) is given by $\Phi(-\mu_c^1/\sqrt{2} - z_\alpha)$, where $\Phi(x)$ is the cumulative distribution function of $N(0, 1)$ and $z_\alpha = \Phi^{-1}(1-\alpha)$ is the $(1-\alpha)$-quantile of that same distribution.

In Fig. 1 we plot the 5% power as a function of the variance of c_i when c_i is drawn from a uniform distribution with mean -5. The variance ranges from 0 to 8, suggesting that the support of c_i varies from -5 to $[-9.9, -0.1]$. There are three curves representing the theoretical local power, and the empirical power for $N = 20$ and $N = 50$ when $T = 500$. Since according to theory power only depends on the mean of c_i, the theoretical power curve is flat. We see that when $N = 20$ the empirical power function is way off the theoretical prediction. It starts at about 7% below (note how the scale of the horizontal axis goes from 75% to 100%) and then the difference just gets larger as the variance of c_i increases. Of course, since in this case N is relatively small the asymptotic approximation is not going to be perfect, and some variations are to be expected. However, the same pattern is observed also when $N = 50$, although the vertical distance to the theoretical power curve is not as large as before. In other words, there seems to be a variance effect at work here that cannot be explained by theory, and that seems to go away, although only very slowly, as N increases.

This example suggests that the asymptotic approach commonly used for analyzing the local power of panel unit root tests may not be sharp enough to capture actual behavior. It is therefore necessary to consider alternative frameworks that are sensible not only asymptotically but also in finite samples, and this paper takes a step in this direction. However, unlike existing works, here the rate of shrinking of the local alternative is not pre-specified but is instead set equal to $\rho_i = 1 + c_i N^{-\kappa} T^{-\tau}$, which includes all previously considered local alternatives as special cases, including the usual time series specification with $\kappa = 0$ and $\tau = 1$ (see, for example, Phillips, 1988). Within this framework, terms that would otherwise be negligible are retained, leading to a more detailed asymptotic analysis. Our main contribution is to show how the moments of some commonly encountered sample quantities can be written as infinite-order (IO) series expansions in the moments of c_i with coefficients that depend on the rate of shrinking of the local alternative, and hence also on κ and τ. The expansions are up to an error that is of order T^{-1}, indicating that they should lead to good approximations even for very small values of N, provided that T is large enough.

The significance of the new moment expansions is that they can be used to obtain IO local asymptotic power functions of any test statistic that depend on the sample quantities considered. A further contribution of this paper is to provide some detailed illustrations of how such a IO local power analysis can be carried out.

As a first illustration we revisit the above example. The results suggest an IO local asymptotic distribution where the mean-only distribution in (1) can be seen as a mere first-order (FO) approximation. Here, the effect of the second moment of c_i is of order $N^{-1/2}$, with subsequent moments in the expansion decaying according to additional powers of $N^{-1/2}$. The results from a small Monte Carlo study show that the new IO theory is very accurate, and that it goes a long way toward explaining the variance effect seen in Fig. 1.

As a second illustration, we consider the test statistics of Moon and Perron (2008), whose main difference lies with how they are adjusted for bias in the presence of incidental intercepts. Our interest in these statistics originates with the fact while it has been observed that the value of κ compatible with non-negligible local asymptotic power depends on the bias adjustment method, little is known as to the working of this dependence (see, for example, Moon and Perron, 2004, 2008; Moon et al., 2006, 2007). The new theory delivers significant insight in this regard. In particular, it shows how different bias adjustment methods cause cancellation of different terms in the moment expansions, and that the observed variation in κ is brought about by a cancellation of the leading term.

In a third illustration, we show how the results provided in the present paper can be used to derive IO results also for test statistics that are not stated directly in terms of the moments considered here, but whose asymptotic distributions can still be obtained from these moments. In particular, the uniformly most powerful test of Becheri et al. (2015) is considered, whose FO local asymptotic distribution is equal to that in (1). By deriving the IO version of this distribution we obtain what might be referred to as an "IO local power envelop".

In our fourth and final illustration, we show how the provided moment expansions, which are derived under many simplifying assumptions, can be used to derive IO local asymptotic distributions also under more general conditions. As an example, we take one of the test statistics previously considered by Moon and Perron (2004), which is general enough to accommodate heteroskedastic, and serial and cross-section correlated errors.

The plan of the paper is as follows. Section 2 lays out DGP considered, which is chosen so as to exclude all distractions but the features that drive local asymptotic power. Section 3 reports the results of the IO expansions of the moments considered, which are put in perspective through a comparison with the corresponding FO moment approximations. Section 4 is concerned with the local power illustrations. Section 5 concludes.

2. Model

The DGP is similar to the one considered in Section 1 and is given by

$$y_{i,t} = \beta_i' d_t^p + u_{i,t}, \tag{2}$$

$$u_{i,t} = \rho_i u_{i,t-1} + \varepsilon_{i,t}, \tag{3}$$

where $u_{i,0} = 0$, $\varepsilon_{i,t}$ is independently and identically distributed (iid) with $E(\varepsilon_{i,t}) = 0$, $E(\varepsilon_{i,t}^2) = \sigma_\varepsilon^2 > 0$ and $E(\varepsilon_{i,t}^4) < \infty$. In the derivations we assume that σ_ε^2 is known (as in, for example, Moon et al., 2007); hence, we can just as well set $\sigma_\varepsilon^2 = 1$. Also, $d_t^p = (1, \ldots, t^p)'$ is a $(p+1)$-dimensional vector of trends, for which we consider three specifications; (i) no deterministic terms ($p = -1$), (ii) incidental intercepts ($p = 0$), and (iii) incidental trends ($p = 1$). Since in practice incidental intercepts are always included, (ii) and (iii) are the empirically most relevant specifications; however, (i) is relevant too, for its simplicity, and we are going to use it here as an illustrative example. We further assume that

$$\rho_i = \exp(N^{-\kappa} T^{-\tau} c_i) = \exp(T^{-1} \lambda_{NT} c_i), \tag{4}$$

where $\lambda_{NT} = N^{-\kappa} T^{1-\tau} < \infty$. The drift parameter c_i is assumed to be iid and independent of $\varepsilon_{j,t}$ for all i, j and t. All the moments of c_i exist, and in what follows it is going to be convenient to denote these as $\mu_c^j = E(c_i^j)$ for $j \geq 1$ and $\mu_c^0 = 0$. The null hypothesis of interest is that of a unit root ($c_1 = \cdots = c_N = 0$), which can be formulated in terms of the moments of c_i as $H_0 : \mu_c^2 = 0$. The relevant alternative hypothesis is given by $H_1 : \mu_c^2 > 0$ (corresponding to $c_i \neq 0$ for some i).

Unlike existing studies where $\tau = 1$ and $\kappa = 1/4$ or $\kappa = 1/2$ is assumed from the outset (see, for example, Breitung, 2000;