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Robust standard errors in transformed likelihood estimation of dynamic panel data models with cross-sectional heteroskedasticity[☆]

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ABSTRACT

This paper extends the transformed maximum likelihood approach for estimation of dynamic panel data models by Hsiao et al. (2002) to the case where the errors are cross-sectionally heteroskedastic. This extension is not trivial due to the incidental parameters problem and its implications for estimation and inference. We approach the problem by working with a mis-specified homoskedastic model, and then show that the transformed maximum likelihood estimator continues to be consistent even in the presence of cross-sectional heteroskedasticity. We also obtain standard errors that are robust to cross-sectional heteroskedasticity of unknown form. By means of Monte Carlo simulations, we investigate the finite sample behavior of the transformed maximum likelihood estimator and compare it with various GMM estimators proposed in the literature. Simulation results reveal that, in terms of median absolute errors and accuracy of inference, the transformed likelihood estimator outperforms the GMM estimators in almost all cases.

1. Introduction

In dynamic panel data models where the time dimension (T) is short, the presence of lagged dependent variables among the regressors makes standard panel estimators inconsistent, and complicates statistical inference on the model parameters considerably. To deal with these difficulties a sizable literature has emerged, starting with the seminal papers of Anderson and Hsiao (1981, 1982) who proposed the application of the instrumental variable (IV) approach to the first-differenced form of the model. More recently, a large number of studies have been focusing on the generalized method of moments (GMM), see, among others, Holtz-Eakin et al. (1988), Arellano and Bond (1991), Arellano and Bover (1995), Ahn and Schmidt (1995) and Blundell and Bond (1998). One important reason for the popularity of GMM in applied economic research is that it provides asymptotically valid inference under a minimal set of statistical assumptions. Arellano and Bond (1991) proposed GMM estimators based on moment conditions where lagged variables in levels are used as instruments. Blundell and Bond (1998) showed that the performance of this estimator deteriorates when the parameter associated with the lagged dependent variable is close to unity and/or the *variance ratio* of the individual effects to the idiosyncratic errors is large, since in such cases the instruments are only weakly related to the lagged dependent variables.[1] The poor finite sample properties of GMM estimators has been documented using Monte Carlo

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1 See also the discussion in Binder et al. (2005), who proved that the asymptotic variance of the Arellano and Bond (1991) GMM estimator depends on the variance of the individual effects.

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studies by Kiviet (2007), for example. To deal with the weak instrument problem, Arellano and Bover (1995) and Blundell and Bond (1998) proposed the use of extra moment conditions arising from the model in levels, which become available when the initial observations satisfy certain conditions. The resulting GMM estimator, known as system GMM, combines moment conditions for the model in first differences with moment conditions for the model in levels. We refer to Blundell et al. (2000) for an extension to the multivariate case, and for a Monte Carlo study of the properties of GMM estimators using moment conditions from either the first differenced and/or levels models. More recently, Bun and Windmeijer (2010) show that the model in levels suffers from the weak instrument problem when the variance ratio is large, and Hayakawa (2007) provides finite sample evidence on the bias of the system GMM estimator for different values of the variance ratio and show that the bias rises with the variance ratio. To overcome these shortcomings, Han and Phillips (2010) and Han et al. (2014) propose alternative GMM estimators.

The GMM estimators have been used in a large number of empirical studies to investigate problems in areas such as labor, development, health, macroeconomics and finance. Theoretical and applied research on dynamic panels have mostly focused on the GMM, and has by and large neglected the maximum likelihood (ML) approach though there are several theoretical advances such as Hsiao et al. (2002), Binder et al. (2005), Alvarez and Arellano (2004), and Kruiniger (2008). Hsiao et al. (2002) propose the transformed likelihood approach while Binder et al. (2005) have extended the approach to estimating panel VAR (PVAR) models. Alvarez and Arellano (2004) have studied ML estimation of autoregressive panels in the presence of time-specific heteroskedasticity (see also Bhargava and Sargan (1983)). Kruiniger (2008) considers ML estimation of a stationary/unit root AR(1) panel data models. More recently, several papers including Han and Phillips (2013), Moral-Benito (2013), Kruiniger (2013), and Juodis (2013) also consider the ML approach to estimating dynamic panel data models. There are several reasons why the GMM approach is preferred to the ML approach. First, the regularity conditions required to prove consistency and asymptotic normality of the GMM type estimators are relatively mild and allow for the presence of cross-sectional heteroskedasticity of the errors. In particular, see Arellano and Bond (1991), Arellano and Bover (1995) and Blundell and Bond (1998). Second, for the ML approach, the incidental parameters problem and the initial conditions problem lead to a violation of the standard regularity conditions, which causes inconsistency. Although Hsiao et al. (2002) developed a transformed likelihood approach to overcome some of the weaknesses of the GMM approach (particularly the weak IV problem), their analysis still requires the idiosyncratic errors to be homoskedastic, which is likely to be restrictive in many empirical applications.[2]

It is therefore desirable to extend the transformed ML approach of Hsiao, Pesaran and Tahmiscioglu (HPT) so that it allows for heteroskedastic errors.[3] This is accomplished in this paper. The extension is not trivial due to the incidental parameters problem that arises, in particular its implications for inference. We follow the time series literature, and initially ignore the error variance heterogeneity and work with a mis-specified homoskedastic model, but show that the transformed maximum likelihood estimator by Hsiao et al. (2002) continues to be consistent. We then derive, under fairly general conditions, a covariance matrix estimator for the quasi-ML (QML) estimator which is robust to cross-sectional heteroskedasticity. Using Monte Carlo simulations, we investigate the finite sample performance of the transformed QML estimator and compare it with a range of GMM estimators. Simulation results reveal that, in terms of median absolute errors and accuracy of inference, the transformed likelihood estimator outperforms the GMM estimators in *almost all* cases when the model contains an exogenous regressor, and in many cases if we consider pure autoregressive panels.

The rest of the paper is organized as follows. Section 2 describes the model and its underlying assumptions. Section 3 proposes the transformed QML estimator for cross-sectionally heteroskedastic errors. Section 4 provides an overview of the GMM estimators used in the simulation exercise. Section 5 describes the Monte Carlo design and comments on the small sample properties of the transformed likelihood and GMM estimators. Finally, Section 6 ends with some concluding remarks.

2. The dynamic panel data model

Consider the following dynamic panel data model

$$y_{it} = \alpha_i + \gamma y_{i,t-1} + \beta x_{it} + u_{it}, \quad i = 1, 2, \dots, N, \tag{1}$$

where α_i, $(i = 1, 2, \dots, N)$ are the unobserved individual effects, u_{it} is an idiosyncratic error term, x_{it} is observed regressor assumed to vary over time (t) and across the individuals (i). It is further assumed that x_{it} is a scalar variable to simplify the notations.[4] We refer to this model as ARX, to distinguish it from the pure autoregressive specification (AR) that does not include the exogenous regressor, x_{it}. The coefficients of interest are γ and β, which are assumed to be fixed finite constants. No restrictions are placed on the individual effects, α_i. They can be heteroskedastic, correlated with x_{jt} and u_{jt}, for all i and j, and can be cross-sectionally dependent. In contrast, the idiosyncratic errors, u_{it}, are assumed to be uncorrelated with $x_{it'}$ for all i, t and t'. However, we allow the variance of u_{it} to vary across i, and let the variance ratio, $\tau^2 = \left[N^{-1}\Sigma_{i=1}^N Var\left(\alpha_i\right)\right] / \left[N^{-1}\Sigma_{i=1}^N Var\left(u_{it}\right)\right]$ to take any positive value. We shall investigate the robustness of the QML and GMM estimators to the choices of τ^2 and γ.

Following the literature we take first differences of (1) to eliminate the individual effects[5]

$$\Delta y_{it} = \gamma \Delta y_{i,t-1} + \beta \Delta x_{it} + \Delta u_{it}, \tag{2}$$

and make the following assumptions:

Assumption 1 (*Initialization*). The dynamic processes (1) have started at time $t = -m$, (m being a positive constant) but only the time series data, $\{y_{it}, x_{it}\}$, $(i = 1, 2, \dots, N; t = 0, 1, \dots, T)$, are observed.

Assumption 2 (*Exogenous Variable*). It is assumed that x_{it} is generated either by

$$x_{it} = \mu_i + \phi t + \sum_{j=0}^{\infty} a_j \varepsilon_{i,t-j}, \qquad \sum_{j=0}^{\infty} \left|a_j\right| < \infty \tag{3}$$

[2] In the application of the GMM approach to dynamic panels, it is generally difficult to avoid the so-called many/weak instruments problem, which is shown to result in biased estimates and substantially distorted test outcomes. See Section 5 for further evidence.

[3] Note, however, that since the transformed ML approach does not impose any restrictions on the individual effects, the errors of the original panel (before differencing) can have any arbitrary degree of cross-sectional heteroskedasticity.

[4] Extension to the case of multiple regressors is straightforward at the expense of notational complexity.

[5] As shown in Appendix A of Hsiao et al. (2002), other transformations can be used to eliminate the individual effects and the QML estimator proposed in this paper is invariant to the choice of such transformations.