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Nonparametric estimation and inference for conditional density based Granger causality measures[☆]

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ABSTRACT

We propose a nonparametric estimation and inference for conditional density based Granger causality *measures* that *quantify* linear and nonlinear Granger causalities. We first show how to write the causality measures in terms of *copula densities*. Thereafter, we suggest consistent estimators for these measures based on a consistent nonparametric estimator of copula densities. Furthermore, we establish the asymptotic normality of these nonparametric estimators and discuss the validity of a local smoothed bootstrap that we use in finite sample settings to compute a bootstrap bias-corrected estimator and to perform statistical tests. A Monte Carlo simulation study reveals that the bootstrap bias-corrected estimator behaves well and the corresponding test has quite good finite sample size and power properties for a variety of typical data generating processes and different sample sizes. Finally, two empirical applications are considered to illustrate the practical relevance of nonparametric causality measures.

1. Introduction

Much research has been devoted to building and applying tests of non-causality. However, once we have concluded that a "causal relation" (in the sense of Granger) is present, it is usually important to assess the strength of this relationship. Only few papers have been proposed to measure the causality between random variables. Furthermore, although the concept of causality is naturally defined in terms of conditional distributions, the estimation of the existing causality measures has been done using parametric mean regression models in which the causal relations are linear. Consequently, one simply cannot use the existing measures to quantify the strength of nonlinear causalities. The present paper aims to propose a *nonparametric* estimation and inference for Granger causality measures. The proposed approach is model-free

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and allows us to quantify nonlinear causalities and the causalities that show up in conditional quantiles as well as higher order conditional moments (such as volatilities, skewness, kurtosis, etc.).

The concept of causality introduced by Wiener (1956) and Granger (1969) constitutes a basic notion for studying dynamic relationships between time series. This concept is defined in terms of predictability at horizon one of a (vector) variable Y from its own past, the past of another (vector) variable X, and possibly a vector Z of auxiliary variables. The theory of Wiener–Granger causality has generated a considerable literature; for review see Dufour and Taamouti (2010). Wiener–Granger analysis distinguishes between three basic types of causality: from Y to X, from X to Y, and instantaneous causality. In practice, it is possible that all three causality relations coexist, hence the importance of finding means to quantify their degree. Unfortunately, causality tests fail to accomplish this task, because they only provide evidence on the presence or the absence of causality, and statistical significance depends on the available data and test power. A large effect may not be statistically significant (at a given level), and a statistically significant effect may not be "large" from an economic viewpoint (or more generally from the viewpoint of the subject at hand) or relevant for decision making. Hence, it is crucial to distinguish between the numerical value of a parameter and its statistical significance (see McCloskey and Ziliak (1996)).

Thus, beyond accepting or rejecting non-causality hypotheses – which state that certain variables do not help forecasting other variables – we wish to assess the magnitude of the forecast improvement, where the latter is defined in terms of some loss function (Kullback distance). Even if the hypothesis of no improvement (non-causality) cannot be rejected from looking at the available data (for example, because the sample size or the structure of the process does allow for high test power), sizeable improvements may remain consistent with the same data. Or, by contrast, a statistically significant improvement – which may easily be produced by a large data set – may not be relevant from a practical viewpoint.

The topic of measuring the causality has attracted much less attention. Geweke (1982, 1984b) introduced measures of causality based on mean-square forecast errors. Gouriéroux et al. (1987) proposed causality measures based on the Kullback information criterion and provided a *parametric* estimation for their measures. Polasek (1994, 2002) showed how causality measures can be computed using the Akaike Information Criterion (AIC) and a Bayesian approach. Dufour and Taamouti (2010) proposed short and long run causality measures based on vector autoregressive and moving average models. The estimation of most existing causality measures has been done based on parametric mean regression models. However, the misspecification of parametric model may affect the structure of the causality between the variables of interest. In addition, the dependence in the mean-regression is only due to the mean dependence, and thus it ignores the dependence that show up in conditional quantiles as well as higher order conditional moments. Finally, as shown in many theoretical and empirical papers, several "causal relations" are nonlinear; see for example Gabaix et al. (2003), Bouezmarni et al. (2012) and Bouezmarni and Taamouti (2011), and references therein. Hence, the existing estimation methods for causality measures cannot be used to quantify nonlinear causalities. An exception is the paper of Zheng et al. (2012) who study linear and nonlinear strength of dependence without making any parametric assumptions on the data. However, their approach only focuses on the dependence in the mean, whereas our approach deals with any type of dependence.

We propose a nonparametric estimator for Granger causality measures that quantify nonlinear causalities and causalities that show up in higher order conditional moments. The nonparametric estimator is model-free and therefore it does not require the specification of the model linking the variables of interest. We write the theoretical Granger causality measures in terms of copula densities. Copula is a tool that fully quantifies the dependence among the variables of interest, and thus it can be used to characterize the conditional probability density based Granger causality that we consider in this paper. So, it seems natural to define the measures of Granger causality in distribution using copulas. An advantage of such an approach is that it allows us to completely separate the marginal structure from the dependence structure. As noted by Chen and Fan (2006), separate modeling of the temporal dependence and the marginal behavior is particularly important when the dependence structure and the marginal properties of a time series are affected by different exogenous variables.

Thereafter, the causality measures are estimated by replacing the unknown copula densities by their nonparametric estimates. The copula densities are estimated nonparametrically using Bernstein polynomials. For i.i.d. data, Sancetta and Satchell (2004) show that, under some regularity conditions, any copula can be represented by a Bernstein copula. Bouezmarni et al. (2010) provide the asymptotic properties of the Bernstein copula density estimator for dependent data. The nonparametric Bernstein copula density estimates are guaranteed to be non-negative. Since the causality measures are defined using the Kullback distance, the non-negativity of the Bernstein estimators avoids having negative values inside the logarithmic function. Furthermore, there is no boundary bias problem when we use the Bernstein estimator, because by smoothing with beta densities the Bernstein copula density does not assign weights outside its support. Chen and Huang (2007) propose a *bivariate* kernel copula estimator based on local linear kernels that also removes the boundary bias. For the review of how to remove boundary bias in nonparametric estimation, see for example Brown and Chen (1999) and Chen (2000).

We establish the asymptotic normality of the proposed nonparametric estimator. This result is used to build tests for the statistical significance of causality measures. The asymptotic normality is achieved by subtracting some bias terms and then rescale the estimator by the proper variance. We also discuss the validity of local smoothed bootstrap that we use in finite sample settings to compute a bootstrap bias-corrected estimator and to perform statistical test for Granger causality measures. A Monte Carlo simulation study reveals that the bootstrap bias-corrected estimator behaves well and that the test has good power for a variety of typical data generating processes and different sample sizes.

Finally, the empirical importance of measuring nonlinear causalities is illustrated. In a first empirical application we quantify the causality between S&P500 Index returns and many exchange rates (US/Canada, US/UK and US/Japan exchange rates). We find that both exchange rates and stock prices could have a significant impact on each other. We also find that the impact of stock returns on exchange rates is much stronger than the impact of exchange rates on stock returns. In a second application we compare the predictive content of dividend–price ratio, volatility index (VIX) and liquidity factor for stock market returns. The results show that both dividend–price ratio and VIX help to predict stock market returns. The comparison of causality measure estimates indicates that VIX has more predictive content than dividend–price ratio. We also find that liquidity factor of Pastor and Stambaugh (2003) does not help to predict the time-series of stock returns.

The plan of the paper is as follows. Section 2 provides the motivation for considering a nonparametric causality measures. Sections 3 and 4 present the theoretical framework which underlies the definitions of causality measures using probability and copula density functions. In Section 5 we introduce a consistent nonparametric estimator of causality measures based on Bernstein polynomial. We also establish the asymptotic distribution of our estimator and discuss the asymptotic validity of a local bootstrap finite sample test. In Section 6 we extend our results to the case where the