ARTICLE IN PRESS

Journal of Econometrics ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Journal of Econometrics

journal homepage: www.elsevier.com/locate/jeconom

Testing for separability in structural equations

Xun Lu [a,*], Halbert White [b]

[a] Department of Economics, Hong Kong University of Science and Technology, Hong Kong

[b] Department of Economics, University of California, San Diego, United States

ARTICLE INFO

Article history:
Available online xxxx

JEL classification:
c12
C14
C31

Keywords:
Nonparametric test
Specification test
Separability
Heterogeneity
Conditional exogeneity
Treatment effects

ABSTRACT

Separability is an important feature of structural equations, as it implies the absence of unobservable heterogeneity of effects and has significant implications for identification and efficiency of estimation. This paper provides a nonparametric test for separability in structural equations. The test is based on a conditional independence test recently developed by Huang et al. (2013), building on consistent procedures of Bierens (1982, 1990) and Stinchcombe and White (1998). The test is easy to implement and achieves $\sqrt{n}$ local power. We apply our test to study interest rate elasticities of loan demand in microfinance and the impact of education on wages.

1. Introduction

Nonseparable structural models have been the focus of increasing attention in the econometric literature.[1] So far, attention has primarily been focused on identification and estimation, with less attention to testing salient features of nonseparable structural relations. In this paper, we provide a nonparametric test for separability in structural relations.

We consider a structural data generating process

$$Y = r(X, U), \tag{1.1}$$

where Y is the response of interest, X an observable treatment or cause of interest, U denotes other causes that may be unobservable, and r is an unknown measurable function. Eq. (1.1) is nonseparable in the sense that we do not assume that r has an additively separable representation. Formally, we test the following separability hypothesis:

$\mathbb{H}_0$: There exist measurable functions r_1 and r_2 such that

$$r(X, U) = r_1(X) + r_2(U) \ \ a.s. \tag{1.2}$$

$\mathbb{H}_A$: $\mathbb{H}_0$ is false.

Separability can substantially simplify identification and estimation,[2] but as Matzkin (2007, p. 5317) notes, economic theory rarely suggests that the unobservable causes U necessarily influence Y in an additive way. As we show, separability is equivalent to the absence of unobservable heterogeneity of treatment effects (when X is binary) or of marginal effects (when X is a continuous variable). That is, separability ensures that unobservables do not interact with observables in determining Y. This may not be plausible in applications.

Further, imposing separability when it is in fact not present can result in serious errors of interpretation and inference. In particular, separability has important implications for identification of "structural" objects. For example, Hahn and Ridder (2011) show that a conditional moment restriction "identifies the ASF (average structural function) *only if* the model is structurally separable in observable covariates and unobservable random errors" (emphasis added). Similarly, Schennach et al. (2012) show that in triangular structural systems, the interpretation of the local indirect least squares (LILS) estimator crucially depends on the separability of the structural equation that determines X. Finally, separability has implications for the efficiency of estimation, especially when X is

* Correspondence to: Department of Economics, Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong. Tel.: +852 2358 7616; fax: +852 2358 2084.

E-mail address: xunlu@ust.hk (X. Lu).

1 See, for example, the discussion and references in Schennach et al. (2012).

2 There is a large literature on estimation and testing of separable statistical models. For example, see Hall and Horowitz (2005) and the references therein. Lewbel (2001) gives conditions under which separable statistical models are compatible with separable structural models of demand.

http://dx.doi.org/10.1016/j.jeconom.2014.04.005
0304-4076/

Please cite this article in press as: Lu, X., White, H., Testing for separability in structural equations. Journal of Econometrics (2014), http://dx.doi.org/10.1016/j.jeconom.2014.04.005

binary. If so, and if the structural equation is separable, there are more efficient estimators for the average treatment effect and the average effect of treatment on the treated. It is thus an important empirical question as to whether the structural equation is separable.

Throughout, we allow X to be endogenous in the sense that X and U may be correlated or otherwise dependent. We impose a conditional form of exogeneity, suitable for identifying certain effects of interest, namely that X and U are independent given covariates Z, as in Altonji and Matzkin (2005) and White and Chalak (2006). This is analogous to (and implies) the unconfoundedness assumption in the treatment effects literature. Our test is straightforward to implement and is based on the conditional independence test of Huang et al. (2013) (HSW), which builds on results in the specification testing literature (Bierens, 1982, 1990; Bierens and Ploberger, 1997; Stinchcombe and White, 1998 (SW)). The test is nonparametric, yet achieves $\sqrt{n}$ local power. The test statistic is based on a distance, suitably measured, between restricted and unrestricted estimators. The distance is evaluated using a class of functions (generically comprehensively revealing (GCR) functions) indexed by nuisance parameters. We use kernel estimators to estimate this distance. Our test statistic is obtained by integrating out the nuisance parameters, yielding Cramer–von Mises type tests. We show that the test statistic converges weakly to a non-standard distribution, specifically an integral of a function of a Gaussian process. Critical values for this test statistic are straightforwardly obtained by subsampling or the bootstrap.

There are so far few tests for separability.[3] To the best of our knowledge, Hoderlein and Mammen (2009, HM) and Heckman et al. (2010, HSU), are the only works so far to propose tests for separability in a structural context. HM is mainly concerned with identification and estimation of local average derivatives in nonseparable models; in discussing extensions of their modeling approach, they briefly discuss a test for separability using conditional quantiles (HM, p. 13–14). Our approach differs from HM, in that our test is a regression-based integrated conditional moment-type test, whereas theirs uses conditional quantile regression. Although HM do not provide detailed power analysis for their test, we show that our test detects local alternatives converging to the null at the rate $n^{-1/2}$. HSU provide a test for separability when X is a binary treatment and there is a classical exogenous instrument. Here we use conditioning instruments (covariates or "control variables"); our test applies to binary, categorical, or continuous treatments. Because there can be no generally optimal test in nonparametric contexts, it is valuable to have a variety of complementary approaches to testing a given hypothesis.

The plan of the paper is as follows. In Section 2, we lay out the framework and motivate the test for separability. In Section 3, we discuss testing separability via conditional independence tests. In Section 4, we propose specific test statistics and examine their asymptotic and finite sample properties. In Section 5, we apply our test to study the interest rate elasticities of demand for loans and the impact of education on wages. Section 6 contains concluding remarks. Mathematical proofs are relegated to the Appendix.

2. Framework and motivation

2.1. The data generating process

We first introduce our data generating process assumption. Throughout, random variables are denoted using uppercase letters and their realizations using lowercase.

[3] There is a large literature on testing for additivity in conditional expectation functions (see for example, Li and Racine, 2007, section 9.1, p. 283 and the references therein). But those tests only involve observable variables. In contrast, we test separability between observables X and unobservables U.

Assumption A.1. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a complete probability space. Let (U, X, Y, Z) be random vectors on $(\Omega, \mathcal{F}, \mathbb{P})$ generated by a structural system, such that U is of dimension k_u, where k_u is a positive (possibly countably infinite) integer; X is scalar; the scalar Y is structurally generated as

$$Y = r(X, U), \qquad (2.1)$$

where r is an unknown measurable function; and Z is of dimension $0 \leq k_z < \infty$. Realizations for X, Y, and Z are observable, and realizations of U may be unobservable.

Here we allow X to be binary, categorical, or continuous. For simplicity, we assume that X is a scalar. It is straightforward to generalize to the vector case. When X is binary, we can define such effects as:

$$\text{Treatment effect (TE)}: \tilde{m}(U) := r(1, U) - r(0, U)$$
$$\text{Average treatment effect (ATE)}: \tilde{\beta} := E\left[\tilde{m}(U)\right]$$
$$\text{Average effect of treatment on the treated (ATT)}:$$
$$\tilde{\gamma} := E\left[\tilde{m}(U) \mid X = 1\right].$$

Here, $r(1, U)$ and $r(0, U)$ are the potential outcomes of receiving and not receiving a treatment, respectively. For a detailed discussion of the linkage between structural equation (2.1) and the potential outcomes framework, see White and Lu (2011). When X is continuously distributed and r is differentiable in its first argument, we can define various analogous effects of interest:

$$\text{Marginal effect (ME)}: m(x, U) := \frac{\partial r(x, U)}{\partial x}$$
$$\text{Average marginal effect (AME)}: \beta(x) := E[m(x, U)]$$
$$\text{Local average response (LAR)}: \gamma(x) := E[m(x, U) \mid X = x].$$

Note that $m(x, U)$, $\beta(x)$, and $\gamma(x)$ are continuous versions of $\tilde{m}(U)$, $\tilde{\beta}$, and $\tilde{\gamma}$ at level x, respectively (Florens et al., 2008). For a detailed discussion of AME and LAR, see, for example, Chamberlain (1984), Blundell and Powell (2003), Altonji and Matzkin (2005), and Bester and Hansen (2009).

The random variables Z do not drive Y. Although Z is not a standard instrument, as it is generally correlated with U, it nevertheless plays an instrumental role in identifying the effects of interest just defined. Specifically, Z is a *conditioning* instrument, in the taxonomy of Chalak and White (2011). To state the key conditional exogeneity assumption, we follow Dawid (1979) and write $\mathcal{X} \perp \mathcal{Y} \mid \mathcal{Z}$ to denote that $\mathcal{X}$ and $\mathcal{Y}$ are independent given $\mathcal{Z}$.

Assumption A.2. X and U are not measurable with respect to the sigma-field generated by Z, and

$$X \perp U \mid Z.$$

When X is binary, A.2 is equivalent to the unconfoundedness assumption in the treatment effects literature, which plays a key role in identifying ATE and ATT. A.2 is also a common assumption for identifying effects in the context of nonseparable structural equations (see, for example, Altonji and Matzkin (2005); White and Chalak (2006), and Hoderlein (2011)). There are several cases in which A.2 is plausible. First, one special case of A.2 is that Z has null dimension, i.e., $X \perp U$, the case of strict exogeneity. This holds when X is randomized. Randomized experiments are widely used for policy evaluation, especially in the field of development economics. A second case in which A.2 is plausible is when Z is a proxy for the unobservable causes U or for common causes of X and U. For example, let X be years of education and Y be wages. U represents other drivers of wages, such as ability, that are unobservable. Education is endogenous, since it may be correlated