ARTICLE IN PRESS

Journal of Econometrics ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Journal of Econometrics

journal homepage: www.elsevier.com/locate/jeconom

Causal discourse in a game of incomplete information

Halbert White[a], Haiqing Xu[b], Karim Chalak[c,*,1]

[a] Department of Economics, University of California, San Diego, United States

[b] Department of Economics, University of Texas, Austin, United States

[c] Department of Economics, Boston College, 140 Commonwealth Avenue, Chestnut Hill, MA 02467, United States

ARTICLE INFO

Article history:
Available online xxxx

JEL classification:
C30
C57
C70

Keywords:
Auction
Causality
Game of incomplete information
Simultaneous equations
Structural equations

ABSTRACT

Notions of cause and effect are fundamental to economic explanation. Although concepts such as price effects are intuitive, rigorous foundations justifying causal discourse in the wide range of economic settings remain lacking. We illustrate this deficiency using an N-bidder private-value auction, posing causal questions that cannot be addressed within existing frameworks. We extend the frameworks of Pearl (2000) and White and Chalak (2009) to introduce *topological settable systems* (TSS), a causal framework capable of delivering the missing answers. Particularly, TSS accommodate choices belonging to general function spaces. Our analysis suggests how TSS enable causal discourse in various areas of economics.

1. Introduction and motivation

Causal discourse – that is, discussion of cause and effect – is fundamental to economic explanation. It appears naturally and unselfconsciously throughout Adam Smith's *An Inquiry into the Nature and Causes of the Wealth of Nations* (1776) and in all the major economic contributions of the nineteenth century (e.g., Mill, 1848; Marshall, 1890) and a good part of the twentieth (e.g., Hicks, 1939; Samuelson, 1947). As twentieth-century economists began to think carefully about systems of structural or simultaneous equations, work began on formalizing notions of causality and structure. Classical efforts in this area include the work of Haavelmo (1943, 1944), Marschak (1950), Simon (1953), Strotz and Wold (1960), and Granger (1969). Unfortunately, no clear consensus emerged from this work. Causal notions remained murky, in part due to the causal paradoxes associated with simultaneity, which nevertheless plays an indispensable role in describing economic phenomena. This lack of clarity contributed to economists tending to avoid not only formal discussion of causality, but even informal discussion, as Hoover (2004) documents. Nevertheless, causal discourse is so central to economics and the social sciences and so intuitive that explicit causal discussion has re-emerged in the last few decades, together with renewed and deep consideration of causal foundations (e.g., Holland, 1986; Heckman, 2005).

This renewal has led to significant advances, particularly in program and policy evaluation.[2] Nevertheless, causal discourse still occupies an ambivalent status in a variety of areas of economics. Although it is intuitive and natural to speak about income effects and price effects and the like, rigorous foundations justifying well-posed discussions of cause and effect in the wide range of settings relevant to economics, including game theory, are still lacking. Intuition can only go so far. Without firm foundations, it is easy to go astray when describing economic theories, when analyzing the identification or estimation of causal effects (particularly in the increasingly sophisticated structures analyzed nowadays), and especially when attempting to draw policy conclusions or economic insight from model estimates. There remains a clear need to find broadly applicable rigorous foundations for causal discourse in economics.

We demonstrate this need for a suitable causal framework using the familiar context of an N-bidder private-value auction, a game of incomplete information (Harsányi, 1967). This game is simple enough to allow straightforward analysis, yet rich enough for us

* Corresponding author. Tel.: +1 617 552 6026; fax: +1 617 552 2308.
E-mail address: chalak@bc.edu (K. Chalak).

1 Our coauthor, Halbert White, passed away after completion of a recent draft of this paper. This manuscript is a slight modification of that draft.

2 See e.g., Heckman (2005), Heckman and Vytlacil (2005), Imbens and Wooldridge (2009), and Heckman (2010).

http://dx.doi.org/10.1016/j.jeconom.2014.04.007
0304-4076/© 2014 Published by Elsevier B.V.

to illustrate all of the central issues by posing a variety of relevant causal questions. As we discuss, these basic questions cannot be addressed using existing causal frameworks. We then provide a framework that delivers the missing answers by introducing *topological settable systems* (thereafter TSS), an extension of the causal frameworks of Pearl (2000) and White and Chalak (2009, "WC"). Applying this framework to the N-bidder private-value auction permits delivering answers to causal questions there, but also suggests how TSS can be applied to support causal discourse in more general games and in other areas of economic inquiry.

In game theory, the lack of formal foundations for causal discourse leads, not surprisingly, either to informal discussion of cause and effect, which may be limited or misleading, or, more commonly, to the avoidance of such discussion altogether. As an example of informal causal discourse in game theoretic settings, consider the Wikipedia entry on *complete information*,[3] where we find the following:

> If a game is not of complete information, then the individual players would not be able to predict the effect that their actions would have on the others players (even if the actor presumed other players would act rationally).

Is this statement correct? Does it even make sense? With only intuition as a guide, it is hard to know. Indeed, this statement raises an array of salient questions about causality for incomplete information games: In what sense is a player's strategy or action causally affected by other players' strategies or actions? How is the ceteris paribus "effect" of bidder j's strategy on bidder i's strategy defined? How do rationality in behavior and belief matter for causal discourse? What is the causal role of Harsányi's (1967) agent types (X), if any? Do the number of players (N) and the distribution of types (F) have effects? If so, how? If not, why not? What are the structural equations here? Specifically, are the simultaneous equations of, e.g., Bayesian–Nash equilibrium structural? How about the equilibrium "reduced form"? Is it structural? In particular, do equilibrium strategies and actions have structural meaning and/or causal content?

Giving sensible answers to these questions requires a suitable causal framework. Outside of economics, the Pearl causal model (PCM; Pearl, 2000, def. 7.1.1, p. 203) has emerged as a leading paradigm for understanding cause and effect. The PCM has been applied usefully to address certain causal inquiries (see e.g. Pearl, 2000; Halpern and Pearl, 2005a,b). In particular, the PCM has been productively applied to game theory, and, specifically, to games of incomplete information. Unfortunately, the PCM does not apply to answer the questions above. In seminal work applying the PCM to games, Koller and Milch (2001, 2003) build on "probabilistic graphical models" (e.g. Pearl, 2000) to introduce Multi-Agent Influence Diagrams (MAIDs) for representing and computing equilibrium in non-cooperative games. However, there is no mention of causality in Koller and Milch's (2001,2003) careful work. Probabilistic graphical models and related PCM notions have also been explicitly applied to incomplete information games by Penalva-Zuasti and Ryall (2003), Jiang and Leyton-Brown (2010), and Wellman et al. (2011), among others. Nevertheless, there are generic limitations of the PCM for causal discourse in games (see WC): among other things, the PCM cannot support causal discourse in games with non-unique equilibrium (see also Halpern, 2000); by ruling out a causal role for "background" variables, the PCM does not permit discussion of the causal role played by structurally exogenous variables, such as agent types, X; and the PCM is not explicit about attributes, i.e. non-varying objects that play a role in characterizing systems. As WC illustrate, the PCM also does not apply to complete information pure- and mixed-strategy static games or to infinitely repeated dynamic games. Thus, the PCM does not provide a satisfactory foundation for game-theoretic causal discourse.

In order to overcome these limitations, WC introduced settable systems, extending and refining the PCM to accommodate features essential to economic analysis: optimization, possibly non-unique equilibrium, and learning, while preserving the structural systems spirit of the PCM. Game theory examples where settable systems apply but not the PCM[4] are complete information pure- and mixed-strategy static games, infinitely repeated dynamic games with complete and perfect information, and fictitious play with continuum strategies. Other examples are static consumer demand optimization, dynamic rational expectations consumer demand optimization, stochastic dynamic optimization of consumption and saving, and adaptive dynamic rational expectations models of perfectly competitive markets, among others.

Despite these many applications, settable systems still do not apply to certain major classes of problems, such as incomplete information games. In these games, players' choices of strategy ("type-contingent plans") can be rather general functions, such as monotone functions; but WC's settable systems only admit function variables belonging to topological spaces homeomorphic[5] to the space of countable sequences of reals, such as Hilbert space (e.g., Anderson and Bing, 1968). WC's settable systems framework cannot handle player choices that are elements of more general function spaces. While it may be natural to speak of "cause" and "effect" in these environments, this discourse remains informal, at best, without an adequate rigorous framework. This paper fills this void by introducing TSS, which permit choices to be elements of general function spaces, providing just the right framework for answering the causal questions posed above. We illustrate TSS by applying them to the familiar N-bidder private-value auction. Further, this application suggests how this framework may be applied not only to more elaborate games and to other areas of economics, but even to other fields. For example, TSS may apply to study causality in the spatial–temporal manifolds used to analyze neural activity in the brain (Roebroeck et al., 2011; Valdés-Sosa et al., 2006; Valdés-Sosa et al., 2011).

The plan of the paper is as follows. Section 2 specifies the details of the N-bidder first-price private-value auction that provides our game-theoretic focus throughout. In Section 3, we introduce basic elements of TSS and relate these to individually rational behavior in the N-bidder first-price private-value auction, resolving several of the causal questions posed above. In Section 4, we introduce further elements of TSS and relate these to Bayesian–Nash equilibrium, resolving the remaining questions. In particular, we introduce the notions of comparable and compatible settable systems and employ these to distinguish mutual consistency conditions from structural equations. Section 5 contains further discussion. Section 6 summarizes and concludes by providing explicit answers to each of the causal questions posed above. An Appendix contains supplementary material.

2. An *N*-bidder private-value auction

We consider the first-price private-value auction studied by Guerre et al. (2000) and others in the empirical auction literature and treated by Krishna (2010, chapter 2). There is a single object for sale, and $1 < N < \infty$ potential buyers bid for the object. The highest bidder pays the amount they bid (first price) and gets

3 http://en.wikipedia.org/wiki/Complete_information.

4 See WC, White et al. (2011b), and Chen and White (1998).

5 Recall that two spaces are homeomorphic if there exists a homeomorphism between them, that is, a one-to-one function continuous in both directions.