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On the network topology of variance decompositions: Measuring the connectedness of financial firms

Francis X. Diebold[a,*], Kamil Yılmaz[b]

[a] University of Pennsylvania, United States

[b] Koç University, Turkey

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ABSTRACT

We propose several connectedness measures built from pieces of variance decompositions, and we argue that they provide natural and insightful measures of connectedness. We also show that variance decompositions define weighted, directed networks, so that our connectedness measures are intimately related to key measures of connectedness used in the network literature. Building on these insights, we track daily time-varying connectedness of major US financial institutions' stock return volatilities in recent years, with emphasis on the financial crisis of 2007–2008.

"When you can measure what you are speaking about, and express it in numbers, you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind: it may be the beginning of knowledge, but you have scarcely, in your thoughts, advanced to the stage of science".
[Kelvin (1891)]

"None of us anticipated the magnitude of the ripple effects".
[Merrill Lynch President Gregory Fleming
on the financial crisis of 2007–2008,
as reported in Lowenstein (2010)]

1. Introduction

Connectedness would appear central to modern risk measurement and management, and indeed it is. It features prominently in key aspects of market risk (return connectedness and portfolio concentration), credit risk (default connectedness), counter-party and gridlock risk (bilateral and multilateral contractual connectedness), and not least, systemic risk (system-wide connectedness). It is also central to understanding underlying fundamental macroeconomic risks, in particular business cycle risk (intra- and inter-country real activity connectedness).

Perhaps surprisingly, then, connectedness remains a rather elusive concept, in many respects incompletely defined and poorly measured. Correlation-based measures remain widespread, yet they measure only pairwise association and are largely wed to linear, Gaussian thinking, making them of limited value in financial-market contexts. Different authors chip away at this situation in different ways. The equi-correlation approach of Engle and Kelly (2012), for example, effectively focuses on average pairwise correlation. The CoVaR approach of Adrian and Brunnermeier (2011) and the marginal expected shortfall (MES) approach of Acharya et al. (2010) and Acharya et al. (2012) go beyond pairwise association, tracking association between individual-firm and overall-market movements, in one direction or the other. The equi-correlation, CoVaR and MES approaches are certainly of interest, but they measure different things, and a unified framework remains elusive.

To address this situation, in this paper we develop and apply a unified framework for conceptualizing and empirically measuring connectedness at a variety of levels, from pairwise through system-wide, using variance decompositions from approximating models.

* Correspondence to: Department of Economics, University of Pennsylvania, 3718 Locust Walk, Philadelphia, PA 19104-6296, United States. Tel.: +1 215 898 1507; fax: +1 215 573 2057.

E-mail address: fdiebold@sas.upenn.edu (F.X. Diebold).

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We are proud and grateful to be able to build upon the pioneering insights of Halbert L. White Jr., in several ways ranging from the general to the specific. Generally, for example, our connectedness measures are very much linked to and built upon his tradition of dynamic predictive modeling under misspecification.[1] Specifically, in addition, our approach is tightly linked to the graphical (i.e., network) models in which he made pioneering contributions to understanding causal linkages.[2]

We proceed as follows. In Section 2 we introduce the conceptual framework and population connectedness measures. In Section 3 we treat connectedness estimation. In Section 4 we relate our framework and connectedness measures to both the network literature and the systemic risk literature; the relationships turn out to be direct and important. Finally, in Section 5, we apply our framework to study connectedness at all levels among a large set of return volatilities of US financial institutions during the last decade, including during the financial crisis of 2007–2008. We conclude in Section 6.

2. Population connectedness

Our approach to connectedness is based on assessing shares of forecast error variation in various locations (firms, markets, countries, etc.) due to shocks arising *elsewhere*. This is intimately related to the familiar econometric notion of a variance decomposition, in which the forecast error variance of variable i is decomposed into parts attributed to the various variables in the system. We denote by d_{ij}^H the ij-th H-step variance decomposition component; that is, the fraction of variable i's H-step forecast error variance due to shocks in variable j. All of our connectedness measures – from simple pairwise to system-wide – are based on the "non-own", or "cross", variance decompositions, d_{ij}^H, $i, j = 1, \ldots, N$, $i \neq j$. The key is $i \neq j$.

2.1. The population data-generating process

Consider an N-dimensional covariance-stationary data-generating process (DGP) with orthogonal shocks: $x_t = \Theta(L)u_t$, $\Theta(L) = \Theta_0 + \Theta_1 L + \Theta_2 L^2 + \cdots$, $E(u_t u_t') = I$. Note that Θ_0 need not be diagonal. All aspects of connectedness are contained in this very general representation. In particular, contemporaneous aspects of connectedness are summarized in Θ_0, and dynamic aspects in $\{\Theta_1, \Theta_2, \ldots\}$. Nevertheless, attempting to understand connectedness via the potentially many hundreds of coefficients in $\{\Theta_0, \Theta_1, \Theta_2, \ldots\}$ is typically fruitless. One needs a transformation of $\{\Theta_0, \Theta_1, \Theta_2, \ldots\}$ that better reveals and more compactly summarizes connectedness. Variance decompositions achieve this.

2.2. The population connectedness table

The simple Table 1, which we call a *connectedness table*, proves central for understanding the various connectedness measures and their relationships. Its main upper-left $N \times N$ block contains the variance decompositions. For future reference we call that upper-left block a "variance decomposition matrix", and we denote it by $D^H = [d_{ij}^H]$. The connectedness table simply augments D^H with a rightmost column containing row sums, a bottom row containing column sums, and a bottom-right element containing the grand average, in all cases for $i \neq j$.

The off-diagonal entries of D^H are the parts of the N forecast error variance decompositions of relevance from a connectedness perspective; in particular, they measure pairwise directional connectedness. Hence we define the *pairwise directional connectedness* from j to i as

$$C_{i \leftarrow j}^H = d_{ij}^H. \tag{1}$$

Note that in general $C_{i \leftarrow j}^H \neq C_{j \leftarrow i}^H$, so there are $N^2 - N$ separate pairwise directional connectedness measures. They are analogous to bilateral imports and exports for each of a set of N countries. Sometimes we are interested in "net", as opposed to "gross", pairwise directional connectedness. We immediately define *net pairwise directional connectedness* as $C_{ij}^H = C_{j \leftarrow i}^H - C_{i \leftarrow j}^H$.[3] There are $\frac{N^2-N}{2}$ net pairwise directional connectedness measures, analogous to bilateral trade balances.

Now consider not the individual elements of D^H, but rather its off-diagonal row or column sums. Take the first row, for example. The sum of its off-diagonal elements gives the share of the H-step forecast error variance of variable 1 coming from shocks arising in other variables (all other, as opposed to a single other). Hence we call the off-diagonal row and column sums, labeled "from" and "to" in the connectedness table, the total directional connectedness measures. That is, we define *total directional connectedness from others to* i as

$$C_{i \leftarrow \bullet}^H = \sum_{\substack{j=1 \\ j \neq i}}^{N} d_{ij}^H, \tag{2}$$

and *total directional connectedness to others from* j as

$$C_{\bullet \leftarrow j}^H = \sum_{\substack{i=1 \\ i \neq j}}^{N} d_{ij}^H. \tag{3}$$

There are $2N$ total directional connectedness measures, N "to others", or "transmitted", and N "from others", or "received", analogous to total exports and total imports for each of a set of N countries. Just as with pairwise directional connectedness, we are sometimes interested in net total effects. We define *net total directional connectedness* as $C_i^H = C_{\bullet \leftarrow i}^H - C_{i \leftarrow \bullet}^H$. There are N net total directional connectedness measures, analogous to the total trade balances of each of a set of N countries.

Finally, the grand total of the off-diagonal entries in D^H (equivalently, the sum of the "from" column or "to" row) measures

Table 1
Connectedness Table Schematic. See Text for details.

	x_1	x_2	$\cdots$	x_N	From others
x_1	d_{11}^H	d_{12}^H	$\cdots$	d_{1N}^H	$\Sigma_{j=1}^N d_{1j}^H, j \neq 1$
x_2	d_{21}^H	d_{22}^H	$\cdots$	d_{2N}^H	$\Sigma_{j=1}^N d_{2j}^H, j \neq 2$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
x_N	d_{N1}^H	d_{N2}^H	$\cdots$	d_{NN}^H	$\Sigma_{j=1}^N d_{Nj}^H, j \neq N$
To others	$\Sigma_{i=1}^N d_{i1}^H$ $i \neq 1$	$\Sigma_{i=1}^N d_{i2}^H$ $i \neq 2$	$\cdots$	$\Sigma_{i=1}^N d_{iN}^H$ $i \neq N$	$\frac{1}{N}\Sigma_{i,j=1}^N d_{ij}^H$ $i \neq j$

[1] See, for example, White (1994).
[2] See, for example, White and Chalak (2009).

[3] We see gross and net connectedness measures as complements, not substitutes, but we sometimes find net measures of interest and sometimes focus on them in our subsequent empirical analysis. Such net measures are precisely analogous to a trade balance, whether bilateral or multilateral – exports of future uncertainty, less imports of future uncertainty – and they are informative and worthy of study, just as is a trade balance in international economics. We hasten to add, of course, that for some purposes one might be interested in examining individual imports and exports, not just their difference.

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