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Identification robust inference in cointegrating regressions

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ABSTRACT

In cointegrating regressions, estimators and test statistics are nuisance parameter dependent. This paper addresses this problem from an identification-robust perspective. Confidence sets for the long-run coefficient (denoted β) are proposed that invert LR-tests against an unrestricted or a cointegration-restricted alternative. For empirically relevant special cases, we provide analytical solutions to the inversion problem. A simulation study, imposing and relaxing strong exogeneity, analyzes our methods relative to standard Maximum Likelihood, Fully Modified and Dynamic OLS, and a stationarity-test based counterpart. In contrast with all the above, proposed methods have good size regardless of the identification status, and good power when β is identified.

1. Introduction

Cointegration models – defined as stationary linear combinations between non-stationary variables – have wide applicability in econometrics. However, it is becoming increasingly clear from the literature that the inference on cointegrating vectors is a challenging problem. In a recent survey, Johansen (2009) discusses, among others, two important reasons for the above. First, cointegrating equations have traditionally been interpreted as long-term relations, yet time series that can be modeled as such are short. Therefore, it becomes a natural part of the methodology to develop finite sample motivated methods. Second, finite sample methods have nevertheless been notably lacking. Available estimators and test statistics heavily rely on asymptotic theory, and more importantly, are nuisance parameter dependent which may cause severe finite sample distortions.

To set focus, consider the vector autoregressive framework of Johansen (1995) which, given a p-dimensional vector X_t, relies on the regression of ΔX_t on X_{t-1}, and e.g. a constant and further lags of ΔX_t. Let Π refer to the coefficient of X_{t-1} in the latter regression. The cointegrating relation and associated long-run coefficient, denoted as the $(p \times r)$ matrix β, are defined in this context via a reduced rank restriction of the form $\Pi = \gamma\beta'$, where r refers to the cointegration rank. This paper focuses on estimating and testing long-run parameters without assuming that they are identified.

Identification failure typically occurs when the statistical objective function does not respond to some parameters, which is inherent to the above structure. This is because β cannot be recovered from the restriction $\Pi = \gamma\beta'$ when γ is close to zero or is rank deficient, so within and close to this region, the likelihood function will inevitably be ill-behaved. Dufour (1997) is perhaps the first to formalize this issue via an illustrative bivariate process.

In traditional discussions of cointegration, related issues with γ are acknowledged although not widely recognized. Johansen (1988, 2000, 2002) show that standard likelihood ratio (LR) criteria are asymptotically χ^2 and Bartlett adjustable as along as $\gamma \neq 0$ yet perform poorly otherwise.[3] Phillips (1994) argues that finite

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3 In fact Johansen (2000, p. 741) defines the problematic parameter subspace as "the boundary where the order of integration or the number of cointegrating relations change".

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sample inference on β is possible in triangular systems setting $\gamma = -(I_r, 0)'$ which amounts to imposing weak exogeneity and ruling out dynamics and feedback.[4] Johansen (1995, Chapter 8) formally links weak exogeneity to zero restrictions on components of γ. Further insights on less restrictive parametrizations of γ and their relevance and implications on inference may be traced back to the simulation design of Gonzalo (1994). One aim of the present paper is to provide an identification basis for understanding and solving such problems.

More generally, identification problems have previously been addressed in a variety of settings including the enduring weak-instruments case.[5] However, to our knowledge, cointegration has not been directly addressed. It may be worth remarking that Dufour (1997) raises yet does not solve the cointegration case. The contribution of the present paper is a formal solution for inference on β placing no prior restrictions on γ. In line with the above cited identification-robust literature, the main principles we follow and show can be summarized as follows. (1) Standard asymptotics provide poor approximations to the distributions of estimators and test statistics. (2) Wald-type confidence intervals of the form {estimate $\pm$ (asymptotic standard error) $\times$ (asymptotic critical point)} will severely understate estimation uncertainty. (3) In contrast, likelihood-ratio type methods admit identification robust bounds which provides a first step towards a useful solution. (4) It is important to consider methods that allow for unbounded and possibly empty outcomes.

A few other papers have considered different although related problems in cointegrating regressions. In particular, Wright (2000) and Muller and Watson (2013) consider models in which regressors have roots local to unity while some linear combination of the regressand and regressors is stationary.[6] Tanaka (1993) and Jansson and Haldrup (2002) define set-ups in which regressors have unit roots yet some linear combination of the regressand and regressors is nearly stationary. Alternatively, Ioannidis and Chronis (2005) assume that nearly integrated series are nearly cointegrated when a linear combination exists with a near integration order that is smaller than the order of near integration of the considered series. With the exception of Wright (2000) and more recently Muller and Watson (2013), this literature does not address inference. Wright (2000) tests a specified value of β by assessing the stationarity of resulting residuals for a single cointegrating vector. Muller and Watson (2013) relax the latter restriction yet work within a common trend definition of cointegration that introduces further complexities via high-dimensional nuisance parameters. Our approach in this papers remains within the tractable and by now well understood reduced rank regression likelihood framework.

Formally, we propose to invert LR-type statistics that test a specified value for β against (i) an unrestricted, or (ii) a cointegration-restricted alternative. Tests on Π in implicit form are also considered as in Phillips (1994). We underscore – as in Wright (2000) – the merits of a confidence set that can be empty, and characterize unbounded outcomes as well. Our results link unbounded and empty confidence sets to departures from the cointegration hypothesis, the consequences of which are of obvious concern. Formally, we show that unbounded confidence sets which suggest that available data is uninformative on β may result from overestimating the rank of Π. In contrast, empty sets may result from underestimating the rank of Π which also reflects departures from the exact unit root assumption on the components of X_t.

Allowing for possible weak identification, we propose three methods to adequately size the above defined statistics. The first method involves a bounds-based critical value; for general insights on the usefulness of bounds when nuisance parameters yield identification problems, see Dufour (1989, 1997), Dufour and Khalaf (2002) and Beaulieu et al. (2013a,b). The latter may be viewed as a *least favorable* (LF) critical value in the sense of Andrews and Cheng (2013). Second, we introduce a data-dependent critical value based on the "*Type* 2 *Robust*" approach from Andrews and Cheng (2013). The latter checks whether available data suggests weak identification and if so, adjusts the cut-off towards the bound via a smooth transition function. Said differently, the Type 2 robust procedure involves a data-based continuous transition from the standard to the bounds-based LF critical value that improves size-corrected power. Third, we examine a simulation-based method based on Dufour (2006) that may be interpreted, because of its parametric basis, as an often unattainable full-information *first best* (FB).

For the special cases $r = 1$ and $r = p - 1$, we provide analytical solutions to the inversion problem. These solutions use the mathematics of quadrics as in Dufour and Taamouti (2005). The proposed LF and Type 2 critical values do not vary with the tested value of β and thus preserve the quadrics form of the test inversion solution for these special cases.

Finally, we conduct a simulation study to assess the properties of our proposed inference methods. In addition, we also check whether and to what degree available competing methods, specifically the Maximum Likelihood of Johansen (1995), the Fully Modified OLS (FMOLS) of Phillips and Hansen (1990) and Phillips (1991, 1995), the Dynamic OLS (DOLS) of Stock and Watson (1993), and the stationarity-test based method from Wright (2000), suffer from identification problems. Our simulation design goes beyond triangular representations that facilitate finite sample analysis; see Gonzalo (1994) or Boswijk (1995) for early references in this regard. We thus follow Gonzalo's simulation design which allows us to control persistence as well as exogeneity. Results can be summarized as follows.

Although high persistence causes size distortions for the considered LR statistics, these are easily corrected as proposed above, imposing and relaxing weak exogeneity. The size of DOLS and FMOLS based t-tests exceeds 90% at the boundary. Furthermore, failure of weak exogeneity causes very severe distortions for DOLS (size $\simeq$88% even with $T = 300$) as well as for FMOLS, albeit to a lesser extent (size nevertheless remains around 37% with $T = 300$), even when β is identified. The test from Wright (2000) is also oversized at the boundary. In contrast, even when weak exogeneity fails, our proposed methods have good size regardless of the identification status, and good power when β is identified. With regards to power, our proposed Type 2 robust method is as powerful as the FB bootstrap. This is noteworthy since the Type 2 method does not require full information, while the FB (here by construction) utilizes the often unavailable information on the dependence structure of residuals in the cointegrating equation.

The remainder of the paper is organized as follows. In Section 2, we set-up the framework and introduce the hypotheses associated with the test we propose to invert. The statistics underlying these tests are defined and analyzed in Section 3, and robust cut-off points are introduced in Section 4. In Section 5, we present the test inversion strategy for the general case. Section 6 discusses the $r = 1$ and $r = p-1$ special cases. The simulation study is discussed in Section 7, while Section 8 concludes the paper. The technical Appendix A.1 summarizes the general projection methods applied, while Appendix A.2 reports the proofs of theorems and lemmas.

4 I_r refers to an r-dimensional identity matrix.

5 See e.g. Dufour (2003), Staiger and Stock (1997), Wang and Zivot (1998), Zivot et al. (1998), Dufour and Jasiak (2001), Kleibergen (2002, 2005), Stock et al. (2002), Moreira (2003), Dufour and Taamouti (2005, 2007), Andrews et al. (2006), Guggenberger and Smith (2008), Antoine and Lavergne (2012), Guggenberger et al. (2012) and Andrews and Cheng (2013).

6 The near unit root issue may be traced back to Stock (1997) and Elliott (1998). See also Zivot (2000), Lanne (2000), Caner and Kilian (2001) and Hjalmarsson and Österholm (2010) and the references therein; on bootstraps with near-unit roots, see e.g. Andrews (2000) and Park (2006).