Journal of Econometrics 185 (2015) 305–331

Contents lists available at ScienceDirect

Journal of Econometrics

journal homepage: www.elsevier.com/locate/jeconom

Residual-based rank specification tests for AR–GARCH type models

Elena Andreou [1], Bas J.M. Werker *

University of Cyprus, Cyprus
and CEPR
Tilburg University, The Netherlands

ARTICLE INFO

Article history:
Received 22 July 2013
Received in revised form
5 November 2014
Accepted 6 November 2014
Available online 22 November 2014

JEL classification:
C22
C32
C51
C52

Keywords:
Conditional heteroskedasticity
Linear and quadratic residual autocorrelation tests
Model misspecification test
Nonlinear time series
Parameter constancy
Residual symmetry tests

ABSTRACT

This paper derives the asymptotic distribution for a number of rank-based and classical residual specification tests in AR–GARCH type models. We consider tests for the null hypotheses of no linear and quadratic serial residual autocorrelation, residual symmetry, and no structural breaks. We also apply our method to backtesting Value-at-Risk. For these tests we show that, generally, no size correction is needed in the asymptotic test distribution when applied to AR–GARCH residuals obtained through Gaussian quasi maximum likelihood estimation. To be precise, we give exact expressions for the limiting null distribution of the test statistics applied to (standardized) residuals, and find that standard critical values often, though not always, lead to conservative tests. For this result, we give simple necessary and sufficient conditions. Simulations show that our asymptotic approximations work well for a large number of AR–GARCH models and parameter values. We also show that the rank-based tests often, though not always, have superior power properties over the classical tests, even if they are conservative. An empirical application illustrates the relevance of these tests to the AR–GARCH models for weekly stock market return indices of some major and emerging countries.

1. Introduction

Recently there has been much interest in specification testing for location–scale time-series models. In this paper we provide a unifying approach to derive the asymptotic distribution of (rank-based) residual specification tests for these models. We focus on the assumptions of independence, symmetry, and stability of innovations in AR–GARCH type models. We also apply our results to Value-at-Risk backtesting. More precisely, we consider the size-correction needed when applying existing and new rank-based tests for: (1) null hypotheses of no linear or (2) no quadratic correlation in the standardized residuals; (3) the null hypothesis of correctly specified innovation quantiles; (4) null hypothesis of symmetry of the innovations; and (5) the null hypothesis of structural stability. The motivation of our special focus on rank-based statistics for the specifications tests lies in the fact that most applications of AR–GARCH models show non-normal innovation distributions (e.g., when applied to financial asset returns).

Specification tests for AR–GARCH type models have a long history. We mention a few examples. Li and Mak (1994) propose a test based on the sample autocorrelation of squared residuals under conditional normality. Berkes et al. (2003) extend this result, dropping the conditional normality condition and using mild non-explosiveness assumptions. Tse (2002) also deals with residual-based specification tests for GARCH type models assuming asymptotically efficient estimators. Lundbergh and Teräsvirta (2002) contributed an important approach that unifies the standard LM-type tests for remaining volatility clustering, as implemented by Bollerslev (1986), and the LM tests of Engle and Ng (1993) for volatility asymmetry. Their approach is quite flexible for checking conditional variance specifications. Halunga and Orme (2009) extend the Lundbergh and Teräsvirta approach to take into account conditional mean estimation uncertainty. Our rank tests for linear and quadratic residual autocorrelation are robust to different innovation distributions and account for the estimation uncertainty. The same holds for the rank-based tests for symmetry and the absence of structural breaks. Our tests turn out to have

* Correspondence to: Econometrics and Finance Group, CentER, Tilburg University, P.O. Box 90153, 5000 LE, Tilburg, The Netherlands.
E-mail addresses: elena.andreou@ucy.ac.cy (E. Andreou), Werker@TilburgUniversity.nl (B.J.M. Werker).

[1] Department of Economics, University of Cyprus, P.O. Box 537, CY1678, Nicosia, Cyprus. Tel.: +357 22 893708; fax: +357 22 895028.

http://dx.doi.org/10.1016/j.jeconom.2014.11.001
0304-4076/

standard limiting distributions and better power for leptokurtic AR–GARCH models compared to, e.g., recently proposed tests in the literature by Kulperger and Yu (2005). The rank-based tests for the null of symmetry of the innovation distribution are easy to implement and, in some cases, compare favorably to other recent tests in the literature including those in Bai and Ng (2001) and Lambert et al. (2012). We find that our rank residual-based tests complement various tests in the literature, are not oversized and enjoy good power properties especially for leptokurtic error innovations of AR–GARCH models. We also apply our method to derive the asymptotic variance of in-sample backtests for Value at Risk (VaR) based on the QML estimator of AR–GARCH models and address the undersizing of some of the regression-based backtests (e.g., Engle and Manganelli, 2004).

The analysis of residual-based tests is generally complicated by its two-step nature. The use of residuals (calculated using *estimated* parameter values) in the test statistics instead of actual innovations may lead to a change in the null limiting distribution and, thereby, to a size-distortion. A well-known example of this phenomenon is the degrees-of-freedom correction when applying the standard Box–Pierce test to residuals of an ARMA-type model. Without this correction, the residual-based Box–Pierce test is undersized. In general, applying a standard test to residuals of some model may lead to both over- and undersizing. Note that in a GMM-type framework, residual-based tests (in the form of a J-test for overidentifying restrictions) will almost automatically always have a limiting null distribution that is less spread-out than the innovation-based tests. We, however, consider a (quasi) likelihood framework that is often applied in the context of AR–GARCH models. In this setting, we show that generally a size correction is needed and, when it is, we provide it explicitly. Often, but unlike the GMM framework not always, ignoring such a correction leads to a conservative, thus still valid, test. We state simple sufficient conditions for the specifications tests we consider to be conservative when applied to residuals.

The present paper has two important theoretical contributions. We give explicit expressions for the limiting null distribution of (rank-based) statistics used in specification testing. We precisely identify situations where the critical values of the tests need not be adjusted, i.e., when the two-step nature of the procedure does not lead to a size distortion. Also, in relevant situations, our results show that the test statistic applied to residuals, but using uncorrected critical values, leads to a conservative test. Thus, in applied work, such tests can be used without adjustment. Our second contribution is that, for the rank-based tests we consider, this conservativeness does not come at the cost of low power. That is, the power of the rank-based tests applied to residuals still makes these tests a competitor for more classical ones. Extensive simulations in Section 4 confirm these asymptotic claims for finite samples and a number of AR–GARCH type specifications.

For our theoretical results, we rely on Andreou and Werker (2012). For another application of their results see Abadie and Imbens (2012). The overhauling framework is that of Locally Asymptotically Normal (LAN) experiments as discussed, e.g., in Bickel et al. (1993), Le Cam and Yang (1990), Pollard (2004), and van der Vaart (1998) and it provides high-level assumptions under which likelihood based inference procedures lead to Gaussian limiting distributions. This method is especially suited for deriving the asymptotic distribution of rank-based statistics since it does not require (asymptotic) smoothness conditions on the statistic of interest with respect to the nuisance parameters. The method also applies to general model specifications, as long as they satisfy the LAN condition. For instance, IGARCH models are not ruled out. In addition, they would also be valid for multivariate GARCH type models which are also LAN and for which our rank-based tests can be readily extended thereby addressing one of the open research questions in such models mentioned in Bauwens et al. (2006). As such, the limiting distributions need not be derived case-by-case in a model specific way.[2]

The rest of the paper is organized as follows. In Section 2 we formally introduce the type of models we consider, including the regularity conditions needed. This leads to our main theoretical result in Theorem 1 and several propositions for special cases. Section 3 derives the asymptotic distribution of three broad categories of specification tests when applied to standardized residuals, namely tests for (1) linear temporal dependence, (2) non-linear temporal dependence, (3) Value-at-Risk backtesting, (4) symmetry, and (5) stability. We use our main theorem to derive the corrections to critical values needed to obtain tests with an exact asymptotic size, and indicate when precisely ignoring such a correction leads to a conservative test. In Section 4 we present a comprehensive simulation study corroborating our theoretical results and showing that the rank-based tests considered in most cases have strong power properties compared to more classical tests widely applied in the literature. Section 5 provides an illustration of our results for modeling some of the major and emerging stock market returns indices. Section 6 concludes the paper.

2. Model and theory

As explained in the introduction, we rely on Andreou and Werker (2012) for the formal analysis of our residual-based specification tests in dynamic location–scale models. That paper provides an analysis which is especially useful for non-pointwise differentiable statistics, for instance, those involving ranks or runs. Their main theorem is based on two assumptions, called ULAN and AN. The ULAN (Uniform Local Asymptotic Normality) condition imposes the model of interest to be sufficiently "regular". For dynamic location–scale models this condition is well-studied and we, thus, refer to existing results. The AN (Asymptotic Normality) condition describes the joint asymptotic behavior of the model's score, the estimator of the unknown model parameters, and the test statistic of interest. This joint limiting variance matrix determines whether a size correction in the residual-based statistic is needed, or not. Both conditions are discussed below in the context of general location–scale time-series models. The most popular application might be for AR–GARCH models, but usually that additional structure does not lead to additional simplification in the expressions.

Consider a time series $Y_1, \ldots, Y_T$ modeled as

$$Y_t = \mu_{t-1}(\eta) + \sigma_{t-1}(\theta)\varepsilon_t, \quad t = 1, \ldots, T, \tag{2.1}$$

where both $\mu_{t-1}(\eta)$ and $\sigma_{t-1}(\theta)$ may depend on past observed values $Y_{t-1}, Y_{t-2}, \ldots$ and, possibly, on exogenous variables X_t. Here, (ε_t) denotes a sequence of i.i.d. innovations. We assume throughout that these innovations have an absolutely continuous density f with finite Fisher information for location and scale, i.e.,

$$\mathcal{I}_\mu = \mathcal{I}_{\mu,f} := \int (f'(x)/f(x))^2 f(x)\mathrm{d}x < \infty, \tag{2.2}$$

$$\mathcal{I}_\sigma = \mathcal{I}_{\sigma,f} := \int (1 + xf'(x)/f(x))^2 f(x)\mathrm{d}x < \infty. \tag{2.3}$$

We also introduce the notation $\mathcal{I}_{\mu\sigma} = \int (f'(x)/f(x))(1 + xf'(x)/f(x))f(x)\mathrm{d}x$, which equals zero in case f is symmetric. Finally, we impose the identification restrictions $\mathrm{E}\varepsilon_t = 0$, $\mathrm{E}\varepsilon_t^2 = 1$, and, also, $\mathrm{E}\varepsilon_t^4 < \infty$. The unknown parameters η and θ are assumed to belong to open Euclidean sets.

Remark 1. The model structure (2.1) is crucial to this paper. The parameter separation where η only occurs in the dynamic mean

[2] Other advances in econometric theory using LAN can be found in Abadir and Distaso (2007), Jeganathan (1995), Ploberger (2004), and Ploberger and Phillips (2012).