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Through the looking glass: Indirect inference via simple equilibria

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ABSTRACT

This paper develops an indirect inference (Gourieroux et al., 1993; Smith, 1993) estimation method for a large class of dynamic equilibria. Our approach consists of constructing econometrically tractable auxiliary equilibria, obtained by simplifying the economic primitives of the structural equilibrium model, via which estimation can proceed. We use this approach to develop an accurate estimator for the long-run risk model of Bansal and Yaron (2004). We demonstrate the method in Monte Carlo simulations and implement it on U.S. data. We also illustrate the good performance of the proposed methodology on an equilibrium model with investor learning.

1. Introduction

The estimation of nonlinear dynamic equilibrium models is the subject of intense research in macroeconomics and finance.[2] Given the technical challenges involved, econometric applications tend to focus on the most tractable equilibria, which impose strong simplifying restrictions on agent preferences, information, and other underlying economic assumptions; we will refer to these tractable cases as *plain vanilla equilibria*. In a distinct branch of the literature, calibration is used to investigate the equilibrium implications of a richer set of economic assumptions; long-run risk in fundamentals, temporal aggregation, incomplete information, or various forms of market frictions have for instance been widely studied. Calibrations suggest that the more general structural models can account for important empirical properties that plain vanilla equilibria cannot capture, but the formal estimation of the general models often remains an open question.

In this paper, we propose that indirect inference is a natural technique for estimating complex structural equilibrium models (Gouriéroux et al., 1993; Smith, 1993) because it permits us to leverage the econometric tractability of closely related plain vanilla equilibria. Indirect inference is a simulation-based method that imputes the parameters of a structural model via an auxiliary model.[3] For a large class of dynamic equilibria, we can naturally construct auxiliary plain vanilla models by simplifying or restricting the set of underlying economic assumptions, while maintaining the condition that observed quantities are in equilibrium. Since the auxiliary and structural models are then closely related, the resulting indirect inference estimator is expected to have good accuracy properties. We illustrate the validity of this intuition on two large-scale examples.

Our first application is the long-run risk (LRR) model of Bansal and Yaron (2004), which has received considerable attention in fi-

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2 See for instance the recent contributions of Dridi et al. (2007), Fernández-Villaverde and Rubio-Ramirez (2007), Fernández-Villaverde et al. (2015), Herbst and Schorfheide (2014), Schorfheide (2013) and Smets and Wouters (2007).

3 Further advances in indirect inference include Calzolari et al. (2004), Czellar et al. (2007), Czellar and Ronchetti (2010), Genton and Ronchetti (2003), Gouriéroux et al. (2010), Heggland and Frigessi (2004), and Sentana et al. (2008).

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nancial economics over the past decade.[4] Under the LRR model, an Epstein–Zin representative agent receives an exogenous consumption stream and prices a stock and a bond. Consumption and dividends are both driven by a small but highly persistent state variable, which is a source of long-run risk. Furthermore, the consumption and pricing data observed by the econometrician are time aggregates corresponding to multiple decision periods of the representative agent. One difficulty with the long-run risk model is that the observation vector is a singular function of the latent state and the likelihood function is exactly zero for uncountably many parameter values. As a result, the likelihood is a highly irregular function of the parameter vector. For these reasons, the estimation of the LRR model has been considered to be especially challenging until now.[5]

The present paper shows that the LRR model can be accurately estimated by indirect inference. We construct a useful auxiliary economy by restricting the agent's utility function and by assuming that the agent's decision frequency coincides with the econometrician's sampling frequency; for every auxiliary economy, the latent state can be fully recovered from the observation vector each period and the likelihood function is available in closed form. We use this property to develop an indirect inference estimator of the Bansal and Yaron (2004) model, which performs well in Monte Carlo simulations.

We estimate the long-run risk model on U.S. quarterly data over the 1947–2008 period. As conjectured by Bansal and Yaron (2004), the long-run risk component is small, highly persistent, and statistically significant. The elasticity of intertemporal substitution has a point estimate of 0.2, and the coefficient of relative risk aversion has a point estimate of 27. The stochastic volatility of consumption exhibits less persistence than is commonly assumed in calibrations. The estimated model captures the basic moments of consumption, dividends, the price-dividend ratio and the riskless rate. A notable exception is that the autocorrelation of dividend growth is negative in the data and positive in the estimated long-run risk model, a discrepancy that confirms the calibration analysis of Beeler and Campbell (2012).

The latent state of the LRR economy can be tracked using an Approximate Bayesian Computation particle filter, a recent advance designed for state-space models in which the probability density of the observation conditional on the state is unavailable in closed-form (Calvet and Czellar, 2011, forthcoming; Jasra et al., 2012). The Approximate Bayesian Computation filter permits us to filter the latent state of the LRR economy and produce forecasts even when the agent's decision frequency differs from the econometrician's observation frequency. We reestimate the LRR model by indirect inference over the 1947–1996 period, and show that the resulting long-run risk model is able to accurately forecast the conditional distribution of aggregate dividend growth and the price-dividend ratio over the 1997–2008 out-of-sample period. By contrast, a calibrated version of the LRR model, or an estimated version with time-invariant volatility, have poor forecasting ability. These results illustrate the practical benefits of the formal estimation method developed in this paper.

Our second application is a general equilibrium model with incomplete information. Sequential learning by economic agents is a powerful mechanism that can theoretically explain key properties of asset returns, aggregate performance and other equilibrium outcomes; see Pástor and Veronesi (2009a) for a survey.[6] Despite their theoretical appeal, however, learning models are not used in practice to forecast and price assets because of a dearth of accurate estimation methods. The present paper contributes to filling this gap.

The complete-information economy, in which the agent fully observes the state of fundamentals, is a natural auxiliary model. We provide sufficient conditions under which the full-information model has a closed-form likelihood. These conditions are satisfied by the examples of Brandt et al. (2004), Calvet and Fisher (2007), David and Veronesi (2006), Lettau et al. (2008), Moore and Schaller (1996) and Van Nieuwerburgh and Veldkamp (2006), and many others. We define the auxiliary estimator by stacking (a) the full-information economy's maximum likelihood estimator and (b) a set of statistics that the incomplete-information model is designed to capture.

We apply our estimation technique to the structural model of daily equity returns developed by Calvet and Fisher (2007). We verify by Monte Carlo simulation that the indirect inference estimator performs well in finite samples. We estimate the structural model on the daily excess returns of the CRSP U.S. value-weighted index between 1926 and 1999. We show that the effect of learning in the return equation is significant, i.e. agents are not fully informed about the state of fundamentals. For the 2000–2009 out-of-sample period, we use the ABC filter to show that the incomplete-information model provides accurate value-at-risk forecasts, which significantly outperform the predictions obtained from historical simulations, GARCH(1,1), and the full-information equilibrium model.

The rest of the paper is organized as follows. Section 2 discusses how to estimate structural equilibrium models by indirect inference. Section 3 applies this method to the long-run risk model of Bansal and Yaron (2004). Section 4 develops an estimator for incomplete-information economies with investor learning. Section 5 concludes. Proofs are included in Appendix A. A supplementary Internet Appendix presents details of estimation methodology.

2. Indirect inference estimation of a dynamic equilibrium model

2.1. Principle

Consider a dataset $y_{1:T} = (y_1', \ldots, y_T')'$ generated from a structural equilibrium model with parameter $\beta^* \in \mathcal{B} \subseteq \mathbb{R}^p$. In order to estimate β^*, we assume that sample paths can be simulated from any model $\beta \in \mathcal{B}$. We also consider that to each structural model $\beta \in \mathcal{B}$, we can associate a closely related auxiliary *plain vanilla* economy that can be efficiently estimated.

The choice of auxiliary economy proceeds as follows. Since the structural model is specified by a set of economic assumptions and equilibrium relationships, one can conveniently define auxiliary economies by modifying the economic primitives of the model

4 See for instance Bansal et al. (2009), Bansal et al. (2005), Bansal et al. (2007), Bansal et al. (2010), Bansal et al. (2012), Bansal and Shaliastovich (2013), Beeler and Campbell (2012), Colacito and Croce (2011), Colacito and Croce (2013), Constantinides and Ghosh (2011), Drechsler and Yaron (2011), Kaltenbrunner and Lochstoer (2010), Hansen et al. (2008), Zviadadze (2013).

5 Bansal et al. (2007) develop a generalized method of moments (GMM) estimator of the long-run risk model. In simulations, however, we have found that the GMM objective function is highly irregular and contains a large number of local optima, and available minimization methods do not produce stable solutions. The explanation is that the recovery of the state requires the inversion of a matrix, which is singular uncountably many times in the general parameter space. By contrast, the estimator of the long-run risk model proposed in this paper is accurate in Monte Carlo simulations and numerically stable on both simulated and empirical data.

6 In financial economics, investor learning has been used to explain phenomena as diverse as the level and volatility of equity prices, return predictability, portfolio choice, mutual fund flows, firm profitability following initial public offerings, and the performance of venture capital investments. In particular, the portfolio and pricing implications of learning are investigated in Brennan (1998), Brennan and Xia (2001), Calvet and Fisher (2007), David (1997), Guidolin and Timmermann (2003), Hansen (2007), Pástor and Veronesi (2009b), Timmermann (1993), Timmermann (1996), Veronesi (1999) and Veronesi (2000).