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The effect of recursive detrending on panel unit root tests☆

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ABSTRACT

This paper analyzes the properties of panel unit root tests based on recursively detrended data. The analysis is conducted while allowing for a (potentially) non-linear trend function, which represents a more general consideration than the current state of affairs with (at most) a linear trend. A new test statistic is proposed whose asymptotic behavior under the unit root null hypothesis, and the simplifying assumptions of a polynomial trend and iid errors are shown to be surprisingly simple. Indeed, the test statistic is not only asymptotically independent of the true trend polynomial, but also is in fact unique in that it is independent also of the degree of the fitted polynomial. However, this invariance property does not carry over to the local alternative, under which it is shown that local power is a decreasing function of the trend degree. But while power does decrease, the rate of shrinking of the local alternative is generally constant in the trend degree, which goes against the common belief that the rate of shrinking should be decreasing in the trend degree. The above results are based on simplifying assumptions. To compensate for this lack of generality, a second, robust, test statistic is proposed, whose validity does not require that the trend function is a polynomial or that the errors are iid.

1. Introduction

1.1. Motivation

Consider the panel data variable $Y_{i,t}$, observable for $t = 1, \ldots, T$ time series and $i = 1, \ldots, N$ cross-section units. One of the main problems in practice when testing for the presence of a unit root in such variables is that the stochastic part of the series cannot be observed directly, but is instead observed subject to some unknown additive trend component. Valid inference on the unit root hypothesis therefore relies critically on the researcher being able to account for the confounding effects of that component. It is therefore common practice to first detrend $Y_{i,t}$, typically by ordinary least squares (OLS), and then to apply the panel unit root test to the resulting detrended data. Two considerations then arise; while underfitting the trend component will result in the test being biased in favor of the unit root null hypothesis, overfitting will inevitably compromise power relative to that obtainable had the correct trend component been chosen. Of course, these considerations are not unique to panel data, but are there also when testing univariate time series. However, if one admits to the possibility of an heterogeneous data generating process (DGP), then the choice of trend component must in principle be made not just once but N times. The introduction of the cross-sectional dimension therefore adds significantly to the complexity of the decision problem.

In time series, the choice of which trend component to fit is often made after considerable consideration to ensure a small number of trend terms, yet still captures the essential features of the trending behavior. This process usually involves some kind of pretesting, such as informal inspection of time plots of the data, and/or testing the significance of the fitted trend coefficients. Interestingly, in panels the choice of trend component is typically much less considerate, and there is almost never any pre-testing involved. A common response to the greater decisional complexity in this case is therefore to simply ignore it.

One reason for why in panels the choice of trend component is given relatively little attention is that most panel unit root tests require that the trend component is the same for all units, and even if this was not the case some kind of overall assessment would seem to be necessary in order to make testing feasible when N is large. Therefore, when viewed within the context of the usual linear trend environment, taking a constant as given, the only deterministic component open to question is a linear trend. The decision

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rule is: include the trend if at least one of the units appears to be trending.

Of course, for many economic time series, a linear trend, rather than a constant, might be considered appropriate as the default specification, and in such cases the deterministic component open for question is a quadratic trend. This is certainly true for series such as GDP, industrial production, money supply and consumer or commodity prices, where trending behavior is evident; however, in panels (omitted) non-linearity is a concern also in general. The reason is that even if the probability that a given series is trending non-linearly might be small, in general it will not be zero, which means that the probability of the panel of multiple time series exhibiting at least some non-linearity will tend to one as N grows.

Unfortunately, unlike in the time series case where the required asymptotic results follow by simple continuous mapping arguments, due to the need to evaluate the moments of functions of the detrended version of $Y_{i,t}$, a (near) random walk process, in panels relaxing the assumption of (at most) a linear trend is far from trivial. In fact, for tests based on OLS detrended data the required moment calculations are basically impossible, except in the special case of a linear trend. Hence, even a researcher recognizing the importance of allowing for non-linear trend terms would run into problems because of the lack of suitable tests.[1]

The use of recursive rather than full-sample OLS detrending holds considerable promise in this regard. The reason is that while in full-sample detrending $Y_{i,t}$ is detrended using the past, current and future values, in recursive detrending only the past and current values are used. Thus, while full-sample detrending destroys the martingale property of the data, recursive detrending preserves it, which is in turn expected to lead to a less biased estimator of the largest autoregressive root (see, for example Shin et al., 2004; Sul, 2009; Chang, 2002, 2012; Demetrescu and Hanck, 2012; Jönsson, 2007; Shin and Kang, 2004). However, while suggestive, none of these studies offer any conclusive theoretical results of the effect of the detrending alone, and most are based exclusively on Monte Carlo simulation (see, for example Shin et al., 2004; Sul, 2009; Jönsson, 2007).[2] Perhaps most importantly, none has recognized the bias-reducing potential of recursive detrending as a means to venture outside the linear trend environment.

Another reason for the relatively inconsiderate treatment of the trend component is that while in time series the "cost" of overfitting is well-known (see Elliott et al., 1996; Hansen, 1995), high power is the very reason for the extension of the unit root methodology to panels in the first place. It has therefore been thought that panel unit root tests are partly robust to the problems of trend overfitting. That is, even if the inclusion of a linear trend might involve overfitting, this should not be detrimental for test performance. Moon et al. (2007) derive the local power envelope for point-optimal tests in the case of at most a linear trend. Their main finding is that the neighborhoods around unity for which the power envelope is defined depend critically on the presence of a linear trend; if the trend is absent, the power envelope is defined within $N^{-1/2}T^{-1}$-neighborhoods of unity, whereas if the trend in present, the power envelope is defined within $N^{-1/4}T^{-1}$-neighborhoods. This is the so-called "incidental trend problem", which has been shown to have a substantial effect on the power of tests based on full-sample OLS detrending (see Moon and Perron, 2004, 2008; Moon et al., 2006, 2007; Westerlund and Larsson, 2012).[3] Hence, contrary to the common belief, the power of panel unit root tests is in fact highly sensitive to the trend degree, and much more so than in time series. This means that the "cost" of achieving invariance with respect to a linear trend is very high, and therefore researchers should take care not to include such a trend unless the data are in fact trending.

In the time series literature it is well known that recursive detrending can be used to avoid the poor power properties of unit root tests based on OLS detrended data (see Shin and So, 2001; Leybourne et al., 2005). The use of recursive detrending in panel data therefore holds considerable promise also in this regard. In spite of this, so far there has been no attempts to study the power implications of recursive detrending from a theoretical point of view. In fact, existing local power studies are based almost exclusively on full-sample OLS detrending (see Moon and Perron, 2004, 2008; Moon et al., 2007; Westerlund and Larsson, 2012).[4] Moreover, as we have already argued, in many instances a linear trend can be taken as given, and the question is whether or not to include a quadratic trend. We therefore ask: what is the "cost" of invariance with respect to higher-order trend terms? A common belief is that the above mentioned increase in the shrinking neighborhood brought about by the inclusion of a linear trend should continue as higher-order trends are added (see Moon et al., 2007, p. 445). This seems reasonable; however, since once outside the linear trend environment nothing is known regarding the power of these tests, we can only speculate. By enabling analysis of more general trend functions, recursive detrending may help to provide an answer to this question.

1.2. Purpose and main findings

The present paper can be seen as a reaction to the above mentioned shortcomings of the existing panel unit root literature. The purpose is to offer an in-depth study of the consequences of recursive detrending in the presence of a (potentially) non-linear trend function. The first test statistic that we consider is based on a simple but transparent DGP in which the trend function is a polynomial and the errors are iid. The polynomial trend specification is interesting because of its wide use and ability to approximate more general trend functions (see, for example Ayat and Burridge, 2000; Harvey et al., 2011). The main findings from the analysis of the first test statistic can be summarized as follows.

As is now well understood, the order of the fitted trend polynomial, which need not be equal to the true one, affects the asymptotic distribution of all unit root test statistics. In time series, this implies that different trend degrees have their own critical values, whereas in panels, it implies that different trend degrees have their own mean and variance correction factors. The test statistic considered in the present paper has the unique and practically very convenient property that its asymptotic null distribution is asymptotically invariant with respect to not only the true but also the chosen trend degree. Hence, unlike existing panel unit root tests, with this test relaxing the linear trend assumption is very simple.

In the local power analysis we consider alternatives that shrink to zero at the rate $N^{-\kappa}T^{-1}$, where $\kappa \geq 0$. Our results show that if the model includes a constant but no trend terms, then the value of κ compatible with non-negligible local power is given by $\kappa = 1/2$, which is the same as for the power envelope in this case. If, on the

1 The lack of a suitable test is also the reason for why in contrast to the time series case where the concern for possible nonlinear trending behavior has led to the development of several procedures for selection the appropriate detrending degree (see, for example Vogelsang, 1998; Ayat and Burridge, 2000; Harvey et al., 2011), in the panel literature there has been no such developments.

2 Specifically, while asymptotic treatments of tests based recursive detrending exist (see Chang, 2002, 2012; Demetrescu and Hanck, 2012; Shin and Kang, 2004), from these studies it is not possible to isolate the effect of the detrending.

3 Moon and Phillips (2000) show that the maximum likelihood estimator of the local-to-unity parameter in near unit root panels is inconsistent. They call this phenomenon, which arises because of the presence of an infinite number of nuisance parameters, an "incidental trend problem", because it is analogous to the well-known incidental parameter problem in dynamic fixed-T panels.

4 The only exceptions are Breitung (2000) and Moon et al. (2006), who consider tests based on Helmert transformed data.