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Decompositions of profitability change using cost functions

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ABSTRACT

The paper presents a decomposition of a production unit's cost ratio over two periods into explanatory factors. The explanatory factors are growth in the unit's cost efficiency, output growth, changes in input prices and technical progress. In order to implement the decomposition, an estimate of the industry's best practice cost function for the two periods under consideration is required. Profitability at a period of time is defined as the value of outputs produced by a production unit divided by the corresponding cost. Using the earlier work by Balk and O'Donnell, the paper provides a decomposition of profitability growth over two periods into various explanatory factors that are similar to the cost ratio decomposition except that change in outputs explanatory factor is replaced by two separate factors: an index of output price growth and a measure of returns to scale.

1. Introduction

In this paper, we will adapt some of the ideas about productivity and profitability decompositions that were explained in Diewert (2011) and O'Donnell (2008, 2010). O'Donnell in his work provided some very useful decompositions that depended on the existence of various output and input quantity aggregates. In the present paper, we suggest that if best practice cost functions are available, then it is "reasonable" to use these best practice cost functions to form aggregates even though some aggregates do not necessarily satisfy all of the axioms that O'Donnell regards as being desirable axioms for a quantity index.[1]

Basically, we will look at the productivity performance of a production unit over two time periods. We assume that we have knowledge of a "best practice" cost function for the production unit for each of the two time periods. This best practice cost function could be the result of a Data Envelopment Analysis exercise or it could arise from an econometric study of the production units in a suitable peer group.[2]

In Section 2, we will use these best practice cost functions to decompose a production unit's cost growth into explanatory factors which are: efficiency growth, changes in output quantities, input price growth and technical progress.

In Section 3, we assume that output prices are available for the production unit in each period and we use these output prices in

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1 See also the discussion in Balk (1998, 90) on desirable axioms for a quantity index in the context of production theory.

2 Alternatively, the reader can simply assume that the production unit is cost efficient in each period and we have an estimated cost function for the unit for each period. In this case, the inequalities in Eqs. (3) and (4) become equalities and the cost efficiency index $\varepsilon(e^0, e^1)$ defined by (5) below is set equal to unity.

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order to form output aggregates. In this section, our focus is on obtaining a decomposition of profitability growth into explanatory factors, where profitability is the ratio of the value of outputs produced in a period to the total cost of producing those outputs. We draw on the cost growth decomposition developed in Section 2 and our main result is the profitability decomposition given by (34). We show how this decomposition is related to similar decompositions that exist in the literature.

Section 4 explores briefly the problem of finding maximum Total Factor Productivity (TFP) output combinations in each period. These TFP maximizing output combinations also maximize profitability for a cost efficient firm.

Section 5 concludes.

2. Decompositions of cost growth into explanatory factors

Before we look at changes in *profitability* (this is the ratio of the value of output in a period to the corresponding value of input)[3] for a production unit (establishment, firm, industry, economy), we will first attempt to obtain a decomposition of the production unit's *cost ratio* into explanatory factors. The decompositions that we develop in this section will prove to be useful in the following section.

Before starting our formal analysis, it is necessary to justify why it is useful to work out a model of production unit efficiency in the context of a cost function model. Many countries have huge public sectors where production units provide goods and services to the public either free of charge or at prices that do not reflect either their costs of production or their desirability to purchasers. However, these public enterprises all hire labor, purchase intermediate inputs and rent (or own) capital equipment and structures. Moreover, information on the prices and quantities of these inputs is generally available in the government or enterprise accounts. If in addition, information on the quantities of outputs produced by these public enterprises is available, then we are in a position to look at the cost efficiency of these production units relative to a peer group of similar units. Under these conditions, it is possible for a productivity analyst to construct a *best practice cost function* that gives the minimum cost of producing the vector of outputs actually produced in each time period by each production unit in the peer group using the most efficient technology that is available to the peer group and using the input prices that the unit faces in the period under consideration. Thus the unit's actual cost in the period can be compared to the corresponding best practice cost (this is the unit's cost efficiency) and we can look at how the unit's cost efficiency evolves over time and decompose this evolution into explanatory factors. This analysis can take place without the analyst knowing output prices. This is the informational setup that will be utilized in the present section. In the next section, we will add the assumption that meaningful output prices are also available to the analyst. However, in the next section, the best practice cost function will continue to play a prominent role in the analysis of the production unit's efficiency over time.[4]

We assume that we can observe the strictly positive *period t input price vector*, $w^t \gg 0_N$, the corresponding N dimensional nonnegative, nonzero *input quantity vector* $x^t > 0_N$[5] for a particular production unit for periods $t = 0, 1$. The *observed cost*, c^t, for the production unit under consideration for period t is:

$$c^t \equiv w^t \cdot x^t; \quad t = 0, 1. \tag{1}$$

We also assume that we can observe the M dimensional nonnegative, nonzero output quantity vector produced by the unit during period t, $y^t > 0_M$ for $t = 0, 1$.

Our next assumption is much stronger than the above assumptions, which are not at all restrictive. We now assume that for each period t, there exists a *best practice technology* that the particular production unit under consideration could potentially access. Thus for each period t, there exists a best practice technology set, S^t, that defines a set of feasible output and input vectors, (y, x), that could be produced in period t if the production unit had access to this technology. In the Appendix, we list some minimal regularity properties that we assume that the sets S^t possess for $t = 0, 1$. We say that there has been *technical progress* between periods 0 and 1 if the production possibilities set S^1 is bigger than the corresponding period 0 set S^0 so that S^0 is a strict subset of S^1. Our final assumption for this section is that we can somehow solve the particular production unit's cost minimization problem for each period using the best practice technology. Thus for $y \geq 0_M$ and $w \gg 0_N$, define the *period t best practice cost function*, $C^t(y, w)$, as follows:

$$C^t(y, w) \equiv \min_x \{w \cdot x : (y, x) \in S^t\}; \quad t = 0, 1. \tag{2}$$

Under our minimal regularity conditions on the production possibilities set S^t, it can be shown that $C^t(y, w)$ will be a nonnegative function, defined for all $y \geq 0_M$ and $w \gg 0_N$, nondecreasing in the components of y for fixed w and concave, continuous, linearly homogeneous and nondecreasing in the components of w for fixed y. If we place stronger regularity conditions on the best practice technology, then $C^t(y, w)$ will satisfy stronger regularity conditions.[6]

In order to implement the decompositions that will be developed in this and subsequent sections, it is necessary that the analyst have estimates of the best practice cost functions, $C^t(y, w)$, for periods 0 and 1. This is possible in the context of a panel DEA study or in the context of estimating an econometric cost function[7] or a stochastic frontier production function using panel data for production units engaged in similar activities.

It will not necessarily be the case that the production unit being studied achieves the best practice level of costs; i.e., the following inequalities will be satisfied:

$$c^t = w^t \cdot x^t \geq C^t(y^t, w^t); \quad t = 0, 1. \tag{3}$$

Thus the observed period t cost for the unit, c^t, will be equal to or greater than the best practice minimum cost, $C^t(y^t, w^t)$, where this minimum cost is computed using the period t best practice technology, the same vector of outputs y^t that the unit produced during period t and facing the same input prices w^t that the production unit faced during period t. Obviously, the difference between these two costs or their ratio can serve as a measure of the cost efficiency of the unit during period t. We will find it convenient to work with the ratio concept and thus we define the *cost efficiency* of the production unit during period t, e^t, as follows[8]:

$$e^t \equiv C^t(y^t, w^t)/w^t \cdot x^t \leq 1; \quad t = 0, 1 \tag{4}$$

where the inequalities in (4) follow from (3). Thus if the establishment or firm is *cost efficient* in period t, e^t will equal its upper bound of 1. Note that the above definition of cost efficiency is equivalent to Farrell's (1957; 255) measure of *overall efficiency* in

3 Balk (2003, 9–10) introduced this terminology.

4 Our contention is that virtually every nonmarket production unit will face market prices for at least some inputs and so it is reasonable to ask the unit to minimize these input costs. It is worth noting that a feature of the analysis presented in this paper is that we do not make any use of distance functions or the concept of technical efficiency by itself (although technical inefficiency can be a part of the cost efficiency concept defined by (4) below). The only functions that we use are the industry best practice cost function and linear functions that use the observed prices and quantities that pertain to a production unit for two periods.

5 Notation: $w \gg 0_N$ means that each element of the N dimensional vector w is positive, $w \geq 0_N$ means that each element of w is nonnegative and $w > 0_N$ means $w \geq 0_N$ but $w \neq 0_N$. The inner product of the N dimensional vectors $w \equiv [w_1, \ldots, w_N]$ and $x \equiv [x_1, \ldots, x_N]$ is denoted as $w \cdot x \equiv \sum_{n=1}^{N} w_n x_n$.

6 Thus if we assume that S^t is a closed convex cone, then $C^t(y, w)$ will be a linearly homogeneous, convex and nondecreasing function of y for fixed w.

7 For an example of such an econometric study, see Lawrence and Diewert (2006).

8 Balk (1998, 28) makes extensive use of this definition.