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Inference on stochastic time-varying coefficient models

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ABSTRACT

Recently, there has been considerable work on stochastic time-varying coefficient models as vehicles for modelling structural change in the macroeconomy with a focus on the estimation of the unobserved paths of random coefficient processes. The dominant estimation methods, in this context, are based on various filters, such as the Kalman filter, that are applicable when the models are cast in state space representations. This paper introduces a new class of autoregressive bounded processes that decompose a time series into a persistent random attractor, a time varying autoregressive component, and martingale difference errors. The paper examines, rigorously, alternative kernel based, nonparametric estimation approaches for such models and derives their basic properties. These estimators have long been studied in the context of deterministic structural change, but their use in the presence of stochastic time variation is novel. The proposed inference methods have desirable properties such as consistency and asymptotic normality and allow a tractable studentization. In extensive Monte Carlo and empirical studies, we find that the methods exhibit very good small sample properties and can shed light on important empirical issues such as the evolution of inflation persistence and the purchasing power parity (PPP) hypothesis.

1. Introduction

This paper introduces a new class of random time varying coefficient (RC) models for bounded non-stationary processes with AR(1) dynamics and proposes kernel-based nonparametric methods for inference on the paths of the unobserved drifting coefficient processes. RC models have been widely discussed in the last few years in applied macroeconomic time series analysis. Work has ranged across topics such as accounting for the Great Moderation, documenting changes in the effect of monetary policy shocks and in the degree of exchange rate pass-through, see e.g. Cogley and Sargent (2001, 2005b), Cogley et al. (2010), Benati (2010), Pesaran et al. (2006), Stock and Watson (1998) and Koop and Potter (2008). It is clear that RC models provide a de facto benchmark technology for analysing structural change. The breadth of the previous work means that the results of this paper have many applications. While kernel based methods form the main approach for estimating models, whose parameters change smoothly and deterministically over time, they have never been considered in the literature as potential methods for inference on RC models, which have been estimated in the context of state space model representations. While the theoretical asymptotic properties of estimating such processes via the Kalman, or related filters are unclear, we show that under very mild conditions, kernel-based estimates of random coefficient processes have very desirable properties such as consistency and asymptotic normality.

The crucial conditions that need to be satisfied to obtain our theoretical results are also commonly imposed for RC models used in applied macroeconomic analysis. These are pronounced persistence of the coefficient process (usually a random walk assumption) coupled with a restriction that the process remains bounded. We formalize these conditions, in a direct intuitive way, while noting that a variety of alternative bounding devices can be used.

The crucial issue of the choice of bandwidth that is perennially present in kernel based estimation is also addressed. We find that a simple choice of bandwidth has wide applicability and can be used irrespective of many aspects of the true nature of the coefficient processes. The latter may have both a deterministic and a stochastic time varying component thus generalizing the two existing polar paradigms. We find that kernel estimation can cope effectively with such a general model and that the choice of bandwidth can be made robust to this possibility.

Although we focus on a simple autoregressive form for the model as a vehicle to investigate our estimator of the unobserved drifting coefficient process, our results are relevant much more widely. They apply to general regression models, multivariate VAR-type models and can be extended to models that allow for

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time-varying stochastic volatility which are used widely in applied macroeconometrics.

The theoretical analysis in this paper is coupled with a extensive Monte Carlo study that addresses a number of issues arising out of our theoretical investigations. In particular, it confirms the desirable properties of the proposed estimators, identified in our theoretical analysis. For example, the theoretically optimal choice of bandwidth is also one of the best in small samples. We illustrate the usefulness of RC modelling in two applications that have received attention in previous work. The first documents changes in inflation persistence over time. The second analyses whether changes in the persistence of deviations from purchasing power parity (PPP) have occurred or not.

The rest of the paper is structured as follows. Section 1.1 discusses the existing literature and provides a framework for our contribution. Section 2 presents the model and some of its basic properties that are of use for theoretical developments. It also contains the main theoretical results on the asymptotic properties of the new estimator. Section 3 provides an extensive Monte Carlo study while Section 4 discusses the empirical application of the new inference methods to CPI inflation and real exchange rate data. Finally, Section 5 concludes. The proofs of all results are relegated to an Appendix.

1.1. Background literature

The investigation of structural change in applied econometric models has been receiving increasing attention in the literature over the past couple of decades. This development is not surprising. Assuming wrongly that the structure of a model remains fixed over time, has clear adverse implications. The first implication is inconsistency of the parameter estimates. A related implication is the fact that structural change is likely to be responsible for most major forecast failures of time invariant models.

As a result a large literature on modelling structural change has appeared. Most of the work assumes that structural changes in parametric models occur rarely and are abrupt. A number of tests for the presence of structural change of that form exist in the literature starting with the ground-breaking work of Chow (1960) who assumed knowledge of the point in time at which the structural change occurred. Other tests relax this assumption. Examples include Brown et al. (1974), Ploberger and Kramer (1992) and many others. In this context it is worth noting that little is being said about the cause of structural breaks in either statistical or economic terms. The work by Kapetanios and Tzavalis (2010) provides a possible avenue for modelling structural breaks and, thus, addresses partially this issue.

A more recent strand of the literature takes an alternative approach and allows the coefficients of parametric models to evolve randomly over time. To achieve this the parameters are assumed to be persistent stochastic processes giving rise to RC models. An early and influential example is Doan et al. (1984) who estimate an RC model on macroeconomic time series and emphasize the utility of Bayesian methods as a way to encode – amongst other things – theoretically informed views that explosive models for data ought to have very low or zero probability. Cogley and Sargent (2005b) deploy an RC model to address the question of whether it was changes in the variance of shocks, or changes in coefficients – policy or otherwise – that gave rise to the period of macroeconomic calmness in the 90s and early 2000s, dubbed the 'Great Moderation'. In this work, and research influenced by it, the authors assume a random walk process for the coefficients of the VAR model, but bound them so that at each point in time the VAR is non-explosive. For the univariate models this amounts to bounding the coefficients between -1 and $+1$. This assumption is justified on the grounds that the monetary authorities would act somehow to ensure that inflation was not explosive. A main point of Cogley and Sargent (2005b) was to respond to criticisms of earlier work (Cogley and Sargent, 2001) that had found evidence of changes in coefficients but without allowing for changes in volatilities, thus potentially biasing their findings in favour of documenting structural change in VAR coefficients. They find evidence of change in the coefficients of the inflation process despite the inclusion of time-varying volatilities. In subsequent work, Cogley et al. (2010) used the same model to investigate whether there had been significant changes in the persistence of inflation (more precisely the gap between inflation and its time varying unobserved permanent component) during the Great Moderation, using the same RC tool. Other examples of the use of this RC tool abound. Benati and Surico (2008) estimate a similar VAR model for inflation and use it to infer that the decline in the persistence of inflation is related to an increased responsiveness of interest rates to deviations of inflation from its target. Mumtaz and Surico (2009) estimate an RC model to characterize evolutions in the term structure and the correspondence of changes therein with the monetary regime. Benigno et al. (2010) estimate a VAR with random walks in the propagation coefficients involving productivity growth, real wage growth and the unemployment rate and find that increases in the variance of productivity growth have a long run effect on the level of unemployment. Researchers have also debated some of the difficulties with the approach. For example, Stock and Watson (1998) discuss how maximum likelihood implementations tend to overstate the probability that the variance of the shock to coefficients is low or zero; Koop and Potter (2008) discuss the difficulty in imposing inequality restrictions on the time-varying autoregressive coefficients, particularly in large dimensional applications and note that it can be hard to find posterior draws that satisfy such conditions.

While the above account gives a clear idea of the current state of the relevant econometric literature, the economic justification of RC models, whose parameters evolve as bounded random walk processes, merits an additional comment. On a practical level, these models are now widely used by empirical macroeconomists. The economic reason for their attractiveness is well explained in the discussion in Cogley and Sargent (2001), who pointed out that fluctuations in parameters of a reduced form economic system may result from evolving beliefs of the policymaker leading to the evolving policy rules. The evolution in beliefs itself is a potential product of the interaction of model misspecification by policymakers and the effects produced by the policy itself, in the economy. Cogley and Sargent (2001) refer to the seminal paper by Lucas (1976), who noted that the practice of macroeconometric modellers of introducing intercept corrections (discussed also in Cooley and Prescott (1973)), may ignore the risk of misspecification due to using a model detached from macroeconomic theory. In Lucas (1976), the author describes how pursuing a policy based on an initial estimation of an atheoretical model would result in time varying reduced form econometric coefficients (not just in intercepts, but more generally). The subsequent work by Sargent (2001) (and later work by Cogley and Sargent (2005a) and Sargent et al. (2006)) was an attempt to describe post war history as the result of a perpetual repetition of the mistakes (or perpetual 'learning') described in the Lucas Critique paper. In summary, the evolution of beliefs and time varying policy in a stochastic economy, may explain why reduced form VAR parameters may evolve themselves as stochastic processes over time. Clearly, the above justification of stochastically varying coefficient models is only one possibility. Equally, time variation may arise because of evolving cultural norms or behaviours, of any agent in the economy, providing a number of other possible avenues for the motivation of RC AR modelling discussed in our work.

A particular issue with the use of such models is the relative difficulty involved in estimating them. As the focus of the analysis