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Estimation for multivariate stable distributions with generalized empirical likelihood

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ABSTRACT

This paper considers the generalized empirical likelihood (GEL) method for estimating the parameters of the multivariate stable distribution. The GEL method is considered to be an extension of the generalized method of moments (GMM). The multivariate stable distributions are widely applicable as they can accommodate both skewness and heavy tails. We treat the spectral measure, which summarizes scale and asymmetry, by discretization. In order to estimate all the model parameters simultaneously, we apply the estimating function constructed by equating empirical and theoretical characteristic functions. The efficacy of the proposed GEL method is demonstrated in Monte Carlo studies. An illustrative example involving daily returns of market indexes is also included.

1. Introduction

The normal distribution has played a central role in the modeling of financial data. However, it is well known that financial data often show asymmetry and heavy-tails properties which cannot be captured by the normal distribution. For this reason, alternative distributions have been investigated.

Elliptical distributions are one of the solutions. Multivariate Student-*t* and multivariate elliptical stable distributions are members of the elliptical distributions, and they can describe the heavy-tailedness. They are applied in financial mathematics such as portfolio theory (e.g., Owen and Rabinovitch (1983)), risk management (e.g., Section 3.3 of McNeil et al. (2005)). Although elliptical distributions are often used for modeling of financial data, they cannot accommodate asymmetry and, hence, they can result in an underestimation of risks. Frahm and Jaekel (2005) introduced the log-returns of the NASDAQ and S&P 500 indexes to illustrate asymmetry in financial data.

To overcome the above difficulties, we consider using multivariate (non-elliptical) stable distributions, which can describe skewed and heavy-tailed distributions. Since Mandelbrot (1963) and Fama (1965) proposed the use of stable distributions to analyze financial data, these distributions have received considerable attention; see, for example, Embrechts et al. (1997), Belkacem et al. (2000), and Rachev and Han (2000). Fig. 1 displays density plots of bivariate normal, elliptical stable, and non-elliptical stable distributions along with contour plots of the densities. The graphs for the non-elliptical stable distribution show skewness that cannot be handled by either normal or elliptical stable distributions. Moreover, the stable distributions arise from a generalization of the central limit theorem in which the assumption of finite variance is relaxed and, consequently, the stable distributions are closed under summation. This property is also a strong motivation to fit stable distributions to financial data, since low-frequency financial returns can be regarded as the sum of high-frequency data.[1]

Despite their appealing properties, estimation in the family of stable distributions is troublesome. The primary difficulties are that, except in a few special cases, the densities have no simple explicit forms and the stable distributions do not necessarily have first or second moments. Although parameter estimation in stable distributions can be challenging, estimation methods for the univariate case have been extensively investigated. DuMouchel (1973) discussed the asymptotic normality of the maximum likelihood estimator, and Nolan (1997, 2001) provided a program for maximum likelihood estimation. Mittnik et al. (1999) studied maximum likelihood estimation by using fast Fourier transforms to approximate the density. Fama and Roll (1971) investigated

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[1] The multivariate skew Student-*t* (e.g., Bauwens and Laurent (2005)) is another distribution which can describe the skewness and heavy-tailedness, but it is not closed under summation.

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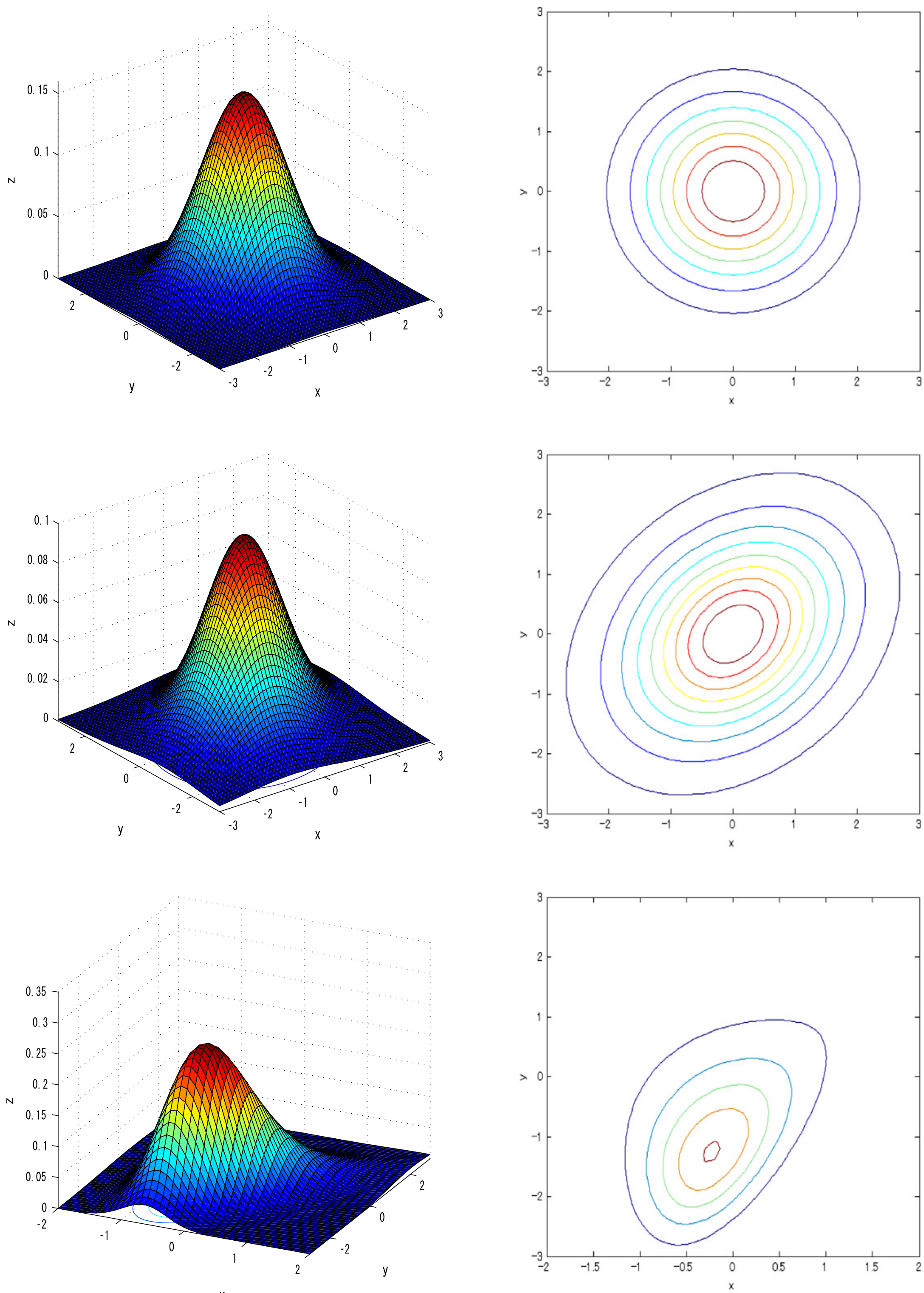

Fig. 1. The left three panels show bivariate density plots and the right three panels show their contours. From top to bottom, the figures are for normal, elliptical stable and non-elliptical stable distributions, respectively.

a quantile-based method for inference in the symmetric case and McCulloch (1986) extended their method to the general case by considering five pre-determined sample quantiles. A method based on characteristic functions was considered by Press (1972), Koutrouvelis (1980, 1981), Kogon and Williams (1998), and Kunitomo and Owada (2004).

Estimation for multivariate stable distributions has also received considerable attention. Using a Monte Carlo EM algorithm, Ravishanker and Qiou (1999) considered estimation for multivariate sub-Gaussian symmetric stable distributions. Lombardi and Veredas (2009) dealt with an indirect estimation method in elliptical multivariate stable distributions. For multivariate stable