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Testing cointegration relationship in a semiparametric varying coefficient model☆

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ABSTRACT

In this paper, we develop two cointegration tests for two varying coefficient cointegration regression models, respectively. Our test statistics are residual based. We derive the asymptotic distributions of test statistics under the null hypothesis of cointegration and show that they are consistent against the alternative hypotheses. We also propose a wild bootstrap procedure companioned with the continuous moving block bootstrap method proposed in Paparoditis and Politis (2001) and Phillips (2010) to rectify severe distortions found in simulations when the sample size is small. We apply the proposed test statistic to examine the purchasing power parity (PPP) hypothesis between the US and Canada. In contrast to the existing results from linear cointegration tests, our varying coefficient cointegration test does not reject that PPP holds between the US and Canada.

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1. Introduction

A great deal of research has been done on testing cointegration relations following the publication of seminal papers by Granger (1987), Engle and Granger (1987) and Johansen (1988). Recently, there is a growing interest in nonlinear, nonparametric and semiparametric cointegrations; see Bierens and Martins (2010), Cai et al. (2009), Choi and Saikkonen (2010), Karlsen et al. (2007), Park and Hahn (1999), Park and Phillips (2001), Wang and Phillips (2009a), Wang and Phillips (2009b) and Xiao (2009).

In the last two decades, a variety of cointegration tests were developed for linear cointegration models. The most popular ones are residual based tests which test whether the estimated residual is a stationary or a unit root process. These tests can be divided into two groups. One is based on the null hypothesis of unit root, which is essentially a unit root test of the estimated residual such as the ADF test used in Engle and Granger (1987) and the Phillips and Ouliaris (1990) tests. Another group is based on the null hypothesis of stationarity, such as the Kwiatkowski et al. (1992) (KPSS) test used in Shin (1994), and the Xiao and Phillips (2002) test. It is natural to consider adapting these traditional cointegration tests for use in nonlinear and nonparametric cointegration frameworks. We find that in general the unit root based cointegration tests do not work in nonparametric kernel estimation situations due to the explosive effects of the estimation errors when there are spurious relations. Thus we turn to the stationarity based tests. We show that the KPSS test still works in the varying coefficient cointegration context, though it has different asymptotics from the asymptotics under the linear cointegration considered in Shin (1994). An adapted version of Xiao and Phillips test was given in Xiao (2009). There is also a group of widely used cointegration tests as Johansen (1991, 1995) which are based on error correction models and could be adapted to certain specified models, see Bierens and Martins (2010).

It should also be mentioned that the estimation and testing of cointegrated relationship in linear regression models have been generalized to various (parametric) nonlinear models. For example, Granger and Terasvirta (1993) considered nonlinear cointegrating relations, and Gregory and Hansen (1996) and Hansen (1992a) studied linear models with non-constant coefficients. Gregory and Hansen (1996) studied the residual-based Z_α-statistic in linear models with regime shifts and a super F statistic was considered by Hansen (1992a) to test a null of cointegrating model against a linear model with random-walk like coefficients.

In this paper, we study cointegration tests of varying coefficient models. Specifically, we consider the following two semiparamet-

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ric varying coefficient models:

$$Y_t = X_t^T \beta(Z_t) + \varepsilon_t, \tag{1}$$

$$Y_t = \beta_1(Z_t) + X_t^T \beta_2(Z_t) + \varepsilon_t, \tag{2}$$

where X_t is a vector of integrated processes of order one ($I(1)$ processes), Z_t is a stationary variable, and $\beta(\cdot)$, $\beta_1(\cdot)$ and $\beta_2(\cdot)$ are some unknown functions with certain smoothness conditions. When ε_t is a stationary process, we say that Y_t and X_t are cointegrated with varying coefficients. If ε_t is an $I(1)$ process, Y_t and X_t are not cointegrated.

We want to test the cointegration relations in models (1) and (2). Our test statistics exploit the KPSS test in the varying coefficient cointegration framework. We derive the asymptotics of our test statistics and show they are similar to those in Shin (1994), and we prove that our test statistics are consistent against the alternatives.

In addition to the well known distortion of KPSS type tests, our simulations show severe distortions for model (2) when the sample size is small. We analyze the reason and propose a wild bootstrap procedure companioned with the continuous moving block bootstrap method proposed in Paparoditis and Politis (2001) and Phillips (2010) to improve the finite sample performance of our tests.

The PPP theory is a cherished part of economics, but empirical support is often found lacking. Theoretically, the logarithm of the nominal exchange rate between two countries should be cointegrated with the logarithm of aggregate price levels of the two countries. However, most empirical findings are contrary to this theoretical prediction. Recently, economists have sought nonlinear explanations of the PPP puzzles based on nonlinear adjustments, e.g., transaction costs, see Taylor and Taylor (2004). We use the tests we propose to test the PPP hypothesis between the US and Canada. Unlike the result of linear cointegration tests, our test suggests that the PPP hypothesis holds between the US and Canada under the varying coefficient cointegration. Our result provides additional encouragement for using the nonlinear approach to resolve the PPP puzzles.

The rest of the paper is organized as follows. In Section 2, we construct our test statistics and examine their asymptotics under the null and the alternative hypotheses. In Section 3, we analyze the reason of the severe distortions of our test statistic for model (2) and provide a wild bootstrap scheme to rectify the problem. Section 4 reports Monte Carlo simulations to examine the finite sample performance of the test statistics. In Section 5, we apply the new test to examine the PPP hypothesis between the US and Canada using monthly data from January 1974 to December 2009. The proofs of main results are relegated to the Appendix.

2. Test statistics and asymptotics

2.1. Varying coefficient cointegration tests

In this paper, we extend the work of testing cointegration relationship to varying coefficient models. We consider the varying coefficient cointegration models (1) and (2). We assume that in the long run integrated series co-move smoothly with respect to some relevant economic variables. If ε_t is a stationary process, models (1) and (2) describe cointegrating relations with smoothly varying coefficients. Consistent estimations of $\beta(\cdot)$, $\beta_1(\cdot)$ and $\beta_2(\cdot)$ can be constructed as in Cai et al. (2009) and Xiao (2009), assuming strict exogeneity in (X_t^T, Z_t) and stationarity of Z_t. Sun et al. (2013) consider the case that both X_t and Z_t are integrated processes.

Models (1) and (2) are quite flexible, and they encompass linear and partially linear regression models as special cases. Thus they can alleviate the potential model misspecification problem associated with the linear regression framework.[1] Cai et al. (2000) considered these models under stationarity. In practice, it is possible that the integrated variables X_t and Y_t are closely related but model (1) suffers from an error-in-variable problem in the sense that one or more relevant $I(1)$ variables are missing from the model, resulting in an $I(1)$ error term ε_t, i.e., we have a spurious regression. Sun et al. (2011) showed that the unknown coefficient $\beta(\cdot)$ in model (1) with ε_t being an $I(1)$ disturbance sequence can still be consistently estimated albeit at a slower convergence rate.

As mentioned above, our test statistics are residual-based cointegration tests motivated by the KPSS test originally proposed to test the null of stationarity. Thus the null and alternative hypotheses are given by

$$H_0 : \varepsilon_t \text{ is a stationary process} \quad \text{vs.} \quad H_1 : \varepsilon_t \text{ is an } I(1) \text{ process.}$$

The tests will base on the following auxiliary model of ε_t.

$$\varepsilon_t = r_t + \eta_t,$$
$$r_t = r_{t-1} + u_t, \tag{3}$$

where η_t and u_t are mean zero stationary processes. Therefore, testing H_0 against H_1 is equivalent to the following

$$H_0 : \sigma_u^2 = 0 \quad \text{vs.} \quad H_1 : \sigma_u^2 > 0. \tag{4}$$

Under H_0, our model describes a long-run cointegrating relation between Y_t and X_t, and the cointegrating coefficients are varying smoothly with respect to another relevant stationary variable Z_t.

Our tests are based on the residuals obtained from estimating the varying coefficient models (1) and (2). Specifically, let $\hat{\varepsilon}_t = Y_t - X_t^T \hat{\beta}(Z_t)$ denote the estimated residual based on the semiparametric model (1), with $\hat{\beta}(Z_t)$ being the semiparametric estimator of $\beta(Z_t)$ from the local pth order polynomial kernel approach, see Section 2.2 for details.

Our test statistic for model (1) is

$$VCC = \frac{n^{-2} h \sum_{t=1}^{n} \left(\sum_{i=1}^{t} \hat{\varepsilon}_i \right)^2}{s_n^2 \kappa^2(K) \varphi_f^2} \tag{5}$$

where $\varphi_f = n^{-1} \sum_{t=1}^{n} (\hat{f}(Z_t))^{-1/2}, \hat{f}(z) = n^{-1} \sum_{s=1}^{n} K_h(Z_s - z)$, $K_h(Z_s - z) = h^{-1} K((Z_s - z)/h)$, $\kappa^2(K) = e_1^T \left(M_p(K)^{-1} R_p(K) \right)^{\otimes 2}$, e_1 is defined in Section 2.2 below, $M_p(K)$ and $R_p(K)$ are constant matrices defined in the Appendix, $A^{\otimes 2} = AA^T$ and $s_n^2 = n^{-1} \sum_{t=1}^{n} \hat{\varepsilon}_t^2$. Here $\hat{f}(z)$ is a consistent estimator of the density function $f(z)$ of Z_t, φ_f is a consistent estimator of $\int \sqrt{f(z)} dz$, $\kappa(K)$ can be calculated for the chosen kernel function, and s_n^2 is a consistent estimator of $E(\eta_t^2)$ under the null hypothesis. We use s_n^2, $\kappa^2(K)$ and φ_f^2 to normalize the test statistic to make it asymptotically pivotal.

It is possible that the disturbance ε_t will fluctuate around a certain level. In the varying coefficient models, this level could also be varying with respect to some variable. For this reason we want to consider model (2). If we test the cointegrating relation in model (2), we first get the estimated residual as

$$\tilde{\varepsilon}_t = Y_t - \tilde{\beta}_1(Z_t) - X_t^T \tilde{\beta}_2(Z_t),$$

where $\tilde{\beta}_1(Z_t)$ and $\tilde{\beta}_2(Z_t)$ are kernel estimators of $\beta_1(Z_t)$ and $\beta_2(Z_t)$ given in Section 2.2. Then the test statistic is given by

$$VCC_\mu = \frac{n^{-2} h \sum_{t=1}^{n} \left(\sum_{i=1}^{t} \tilde{\varepsilon}_i \right)^2}{\tilde{s}_n^2 \kappa^2(K) \varphi_f^2} \tag{6}$$

[1] For more discussions and examples, see Hastie and Tibshirani (1990), and West et al. (1985).