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Testing for seasonal unit roots by frequency domain regression[☆]

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ABSTRACT

This paper develops univariate seasonal unit root tests based on spectral regression estimators. An advantage of the frequency domain approach is that it enables serial correlation to be treated non-parametrically. We demonstrate that our proposed statistics have pivotal limiting distributions under both the null and near seasonally integrated alternatives when we allow for weak dependence in the driving shocks. This is in contrast to the popular seasonal unit root tests of, among others, Hylleberg et al. (1990) which treat serial correlation parametrically via lag augmentation of the test regression. Our analysis allows for (possibly infinite order) moving average behaviour in the shocks. The size and power properties of our proposed frequency domain regression-based tests are explored and compared for the case of quarterly data with those of the tests of Hylleberg et al. (1990) in simulation experiments.

1. Introduction

This paper considers testing for *seasonal* unit roots in a univariate time-series process. In the seminal paper in the literature, Hylleberg et al. (1990) [HEGY] develop separate regression-based t- and F-tests for unit roots at the zero, Nyquist and annual (harmonic) frequencies in the context of quarterly data. Recently, Smith et al. (2009) have generalised this approach to allow for an arbitrary seasonal aspect, while Rodrigues and Taylor (2007) develop near-efficient versions of the HEGY tests. Other important extensions of the basic HEGY approach appear in, *inter alia*, Ghysels et al. (1994), who allow for joint testing across different frequencies, Smith and Taylor (1998), who extend the range of deterministic specifications allowed in HEGY and provide limiting null distributions for the original HEGY statistics, and Rodrigues and Taylor (2004) who develop expressions for the asymptotic local power of the HEGY tests.

These HEGY-type tests are all characterised by the use of parametric lag augmentation, along the lines of the augmented Dickey–Fuller [ADF] test, to allow for weak dependence in the shocks. Focusing on the case where the shocks follow a finite-order autoregressive process of order p [AR(p)], Burridge and Taylor (2001) and Smith et al. (2009) show that such lag augmentation can provide only a partial solution with the limiting null distributions of certain of the harmonic frequency unit root tests still depending, in general, on the parameters of the AR(p) polynomial with the consequence that not all of the HEGY-type tests can be reliably used in practice.

It has been known since the seminal work of Box and Jenkins (1976) that seasonally observed time series tend to display significant moving average behaviour. Indeed Box and Jenkins (1976) developed the well-known seasonal ARIMA factorisations, the best known example of which being the so-called *airline model*. ARMA behaviour can also be a manifestation of neglected periodic autoregressive (PAR) behaviour (see, for example, Ghysels and Osborn, 2001, Chapter 6). As an example, the first-order stationary PAR process for a series observed with period S, admits a stationary and invertible ARMA representation with an MA($S-1$) component. It is therefore important that any seasonal unit root test can allow for moving average behaviour. Recently, del Barrio Castro et al. (2012) have demonstrated that the results of Burridge and Taylor (2001) and Smith et al. (2009) carry over to the case where the shocks admit a stationary and invertible ARMA representation, provided the lag augmentation length increases at any appropriate rate with the sample size, analogous to the results obtained for the ADF test by Said and Dickey (1984).

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Motivated by these issues, the purpose of this paper is to develop a new class of regression-based seasonal unit root tests, which are asymptotically valid in the presence of general weak dependence. Our analysis explicitly allows for the presence of ARMA shocks which need not be invertible but must satisfy the weaker condition that they do not admit spectral zeros at either the zero or seasonal frequencies. We do so by the use of frequency domain regression [FDR] based test statistics. We consider a variety of possible forms for the deterministic component, proposing tests based on both ordinary least squares [OLS] and quasi-difference [QD] de-trending. We demonstrate that the limiting distributions of all of the resulting HEGY-type t- and F-statistics are pivotal under both the null hypothesis and under near-integrated alternatives, attaining the limiting distributions achieved by their standard HEGY counterparts when the shocks are independent and identically distributed [IID].

Frequency domain analysis has a long history in econometrics, with Granger and Hatanaka (1964) providing an early demonstration of its relevance in the analysis of economic data. Furthermore Granger (1966) observed that many economic time series have considerable power at low frequencies, giving rise to a spectral density that is peaked at the origin and which declines as frequency increases; he described this as the typical spectral shape of an economic variable, and the peak at the origin would nowadays be associated with the variable being integrated of order one. In a seasonal setting these peaks occur at the seasonal frequencies, and our approach is based on a seasonal extension of the unit root tests of Choi and Phillips (1993) which utilise the efficient FDR estimator of Hannan (1963). The main advantage of the FDR approach from our perspective is that, unlike the HEGY approach, it delivers estimators of the parameters corresponding to the seasonal roots whose limiting distributions are free from nuisance parameters, even in the presence of moving average disturbances.

The FDR effectively transforms serial correlation in the disturbances into a form of heteroskedasticity across frequencies that is captured by the spectral density function; the resulting estimators handle this heteroskedasticity by weighting the periodogram ordinates by the inverse of the estimated spectral density. In our implementation of the frequency domain estimator we consider two types of spectral density estimator. The first is a simple weighted periodogram estimator [WPE] that averages a set of periodogram values at frequencies either side of the frequency of interest while the second uses the Berk (1974) autoregressive spectral density estimator [ASDE] derived from an autoregressive approximation to the series of interest. Our use of the ASDE is novel in the sense that we use the autoregressive approximation to obtain an estimator of the spectral density across *all* frequencies. This contrasts with its usual use in unit root testing where it is computed at the fixed frequency of the root being tested; see, e.g., Ng and Perron (1995) for the zero frequency root and Rodrigues and Taylor (2007) for the seasonal frequencies.

The paper is organised as follows. Section 2 outlines the seasonal framework, defines the hypotheses of interest, and briefly reviews the HEGY tests. In Section 3 we introduce our FDR implementations of the HEGY statistics and provide representations for their limiting distributions under both the null and local alternatives, showing these to be pivotal in the presence of weak dependence. An investigation into the relative finite sample performances of the FDR tests and the augmented HEGY tests is provided in Section 4. Section 5 concludes. Proofs are contained in Appendices A and B.

2. The seasonal unit root framework

2.1. The seasonal model

The model we consider for the scalar random variable X_t is given by

$$X_t = Y_t + \mu_t, \quad t = 1-S, \dots, T, \tag{2.1a}$$

$$a_S(L)Y_t = U_t, \quad t = 1, \dots, T \tag{2.1b}$$

where $a_S(z) := 1 - \sum_{j=1}^{S} a_j z^j$, S denotes the number of seasons, L denotes the lag operator, and the deterministic component μ_t satisfies

$$\mu_t := \sum_{j=1}^{S} \delta_j D_{jt} + \rho t, \quad t = 1-S, \dots, T, \tag{2.2}$$

where D_{jt} is a seasonal dummy variable such that for $j = 1, \dots, S$, $D_{jt} = 1$ ($t = j \bmod S$) and $D_{jt} = 0$ otherwise. We assume that the random disturbance U_t in (2.1b) is a mean-zero covariance stationary (linear) process satisfying the following conditions:

Assumption 1. The random disturbance U_t in (2.1b) admits the moving average representation $U_t = \psi(L)V_t$ where V_t is $IID(0, \sigma^2)$ with finite fourth moments and where the lag polynomial $\psi(z) := 1 + \sum_{i=1}^{\infty} \psi_i z^i$ satisfies: (i) $\psi(\exp\{\pm i2\pi k/S\}) \neq 0$, $k = 0, \dots, \lfloor S/2 \rfloor$, where $\lfloor \cdot \rfloor$ denotes the integer part of its argument and where $i := \sqrt{-1}$, and (ii) $\sum_{j=1}^{\infty} j|\psi_j| < \infty$.

Remark 1. Assumption 1 ensures that the spectral density function of U_t is bounded, and that it is strictly positive at both the zero and seasonal spectral frequencies, $\omega_k := 2\pi k/S$, $k = 0, \dots, \lfloor S/2 \rfloor$.

The model depicted in (2.1)–(2.2) is sufficiently general to enable X_t to be defined in terms of an arbitrary seasonal frequency S and to capture a variety of seasonal intercept and trend effects in the deterministic component $\mu_t := \gamma' d_t$. We shall consider the following five specifications for the deterministic component in which the stated restrictions on δ_j and ρ hold for $j = 1, \dots, S$:

Scheme 1. No intercept, no trend: $\delta_j = \rho = 0$.
Scheme 2. Intercept, no trend: $\delta_j = \delta$, $\rho = 0$; $\gamma := \delta$, $d_t := 1$.
Scheme 3. Seasonal intercepts, no trend: δ_j unrestricted, $\rho = 0$; $\gamma := (\delta_1, \dots, \delta_S)'$, $d_t := (D_{1t}, \dots, D_{St})'$.
Scheme 4. Intercept, trend: $\delta_j = \delta$, ρ unrestricted $\gamma := (\delta, \rho)'$, $d_t := (1, t)'$.
Scheme 5. Seasonal intercepts, trend: δ_j, ρ unrestricted; $\gamma := (\delta_1, \dots, \delta_S, \rho)'$, $d_t := (D_{1t}, \dots, D_{St}, t)'$.

Smith et al. (2009) also consider the further scheme of seasonal intercepts and seasonal trends,

$$\mu_t := \sum_{j=1}^{S} \delta_j D_{jt} + \sum_{j=1}^{S} \rho_j D_{jt} t, \quad t = 1-S, \dots, T, \tag{2.3}$$

with δ_j and ρ_j unrestricted. Here $\gamma := (\delta_1, \dots, \delta_S, \rho_1, \dots, \rho_S)'$, $d_t := (D_{1t}, \dots, D_{St}, D_{1t}t, \dots, D_{St}t)'$. We will not explicitly cover this case in what follows (as its empirical relevance is limited) but we will mention how our results carry over to this scheme at appropriate points.

In order to simplify our presentation, the initial conditions, $Y_{1-S}, \dots, Y_0$, in (2.1b) are taken to be of $o_p(T^{1/2})$, such that they are asymptotically negligible; cf. Elliott et al. (1996) and Rodrigues and Taylor (2007). Weakening this assumption to allow the initial conditions to be of $O_p(T^{1/2})$ will not alter the null distributions of the tests we outline in this paper provided these are based on