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The aggregation of dynamic relationships caused by incomplete information

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ABSTRACT

We consider the aggregation of heterogeneous dynamic equations across a large population, as introduced by Granger (1980), where the dynamics arise because agents face a signal extraction problem caused by incomplete information. This weakens the independence assumptions used previously in the aggregation literature. We show that, under plausible assumptions, the differenced cross-section aggregate shows long term persistence even though every individual micro-series follows a random walk. As an example, estimates of the model's micro-relations are made using US household panel data.

1. Introduction

1.1. Aggregation and long memory

Among the many seams opened in economics by Sir Clive Granger, one of the richest has been the area of fractional integration. In a ground breaking paper, Granger and Joyeux (1980) studied the properties of a series, Y_t, $t = 0, 1, 2, 3, \ldots$, given by

$$(1-L)^d Y_t = \epsilon_t, \quad (1)$$

where ϵ_t is a white noise process and d, the order of integration of Y_t, is not an integer. The process Y_t is described as $I(d)$. The fractional differencing filter in (1) was also developed and studied independently by Hosking (1981).

This natural extension of the unit root leads to the property of long memory[1]: hyperbolic decay of the autocorrelation function. For the same series this takes the form

$$\gamma_j \equiv Cov(Y_t, Y_{t-j}) \sim cj^{2d-1}, \quad \text{as } j \to \infty, \quad (2)$$

for some constant c. The frequency domain analogue is that the spectral density of the series around the origin is characterised by

$$f(\lambda) \sim c|\lambda|^{-2d}, \quad \text{as } \lambda \to 0, \quad (3)$$

where λ denotes frequency. The parameter d reflects the degree of memory of the series. The autocovariances are not summable for $d > 0$. Despite this, a process is second order stationary, having first and second moments that are stable through time, if $-1/2 < d < 1/2$. Whereas, if $1/2 \le d < 1$, the process is mean reverting but has unbounded variance because the impact of shocks decays at too slow a rate. The singularity resulting from (1) dominates the spectral density around the zero frequency, even when ϵ_t is a finite order auto-regressive moving average (ARMA) process. This has enabled a range of semi-parametric estimators of d, such as those proposed by Geweke and Porter-Hudak (1983) and Robinson (1994), which are consistent, asymptotically normal and robust to short-run dynamics. These estimators have been applied in a number of empirical studies to test for the presence, and to estimate the degree, of long memory in macroeconomic variables. Amongst them, Gil Alana and Robinson (2001) test for fractional integration at seasonal frequencies in quarterly data for consumption and income in the UK and Japan: the same data sets used by Hylleberg et al. (1990) for the initial application of their test for seasonal unit roots. They find evidence to support a non-integer order of integration at some seasonal frequencies, despite being unable to reject the null hypothesis of a unit root at the zero frequency in the UK data. Chambers (1998), also looking at UK aggregate consumption data, finds a value of d of around 1.4.

The fact that shocks persist in long memory models for far longer than in conventional ARMA models has made them useful in modelling a range of economic variables, see Baillie (1996) for an overview of applications in economics and finance. Although fractionally integrated models are not the only models to produce

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1 See Guegan (2005) for a thorough overview of definitions of the term.

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long memory, see Granger and Ding (1996), they have proved very popular in both empirical and theoretical work. Part of this popularity is because they have some theoretical underpinning: they can be generated by the aggregation of heterogeneous dynamic (AR(1)) relationships. Suppose that

$$x_{i,t} = \theta_i x_{i,t-1} + \psi_i \epsilon_t + \upsilon_{i,t}, \tag{4}$$

where ϵ_t is a white noise shock that is common to all agents and $\upsilon_{i,t}$ is a white noise shock that is specific to household i, with $E\left\{\upsilon_{i,t}\upsilon_{j,t}\right\} = 0, \forall i \neq j$. These shocks are held to be independent of one another and the autoregressive parameter, θ_i. The properties of the aggregate $X_{N,t} = \sum_{i=1}^{N} x_{i,t}$ for large N were considered by Robinson (1978). While independently Granger (1980) showed that as the number of relationships to be aggregated becomes large, the aggregate can exhibit long memory. In that paper, the autoregressive parameter varies on (0, 1) with a type 2 beta density

$$f(\theta) = \frac{2}{B(p,q)}\theta^{2p-1}(1-\theta^2)^{q-1}, \quad p, q > 0,$$
$$0 \le \theta \le 1,\ f(\theta) = 0,\ \text{elsewhere},$$

with $B(p, q) = \Gamma(p)\Gamma(q)/\Gamma(p+q)$ where $\Gamma(q) = \int_0^\infty y^{q-1}e^{-y}dy$ is the Gamma function. The memory parameter d is always less than one and depends negatively on the parameter q, which determines the shape of the density approaching one.

The aggregation of heterogeneous AR(1) relationships has also been studied by Gonçalves and Gourieroux (1998), who develop Granger's analysis and consider the behaviour of common and idiosyncratic components when the aggregate is an average across the population. They establish what has become known as the 'usual result' that idiosyncratic shocks tend to disappear from the aggregate asymptotically as the population gets larger. Pesaran (2003) also considers this problem in the context of establishing an optimal aggregate forecast function that minimises in mean square the error in forecasting the aggregate based on past and current values of aggregate variables alone. He shows that the optimal forecast function cannot be represented as a finite order distributed lag model and, in a simulation, that estimates of population parameters are biased downwards when a representative agent model is estimated with aggregate data, but not when the optimal forecast function is estimated. Lippi and Zaffaroni (1998) and Zaffaroni (2004) provide a rigorous analysis of cross-sectional aggregation under weaker assumptions and, significantly, establish the conditions under which idiosyncratic shocks do not tend asymptotically to zero, although they always do after differencing. Oppenheim and Viano (2004) consider the aggregation of processes with roots at non-zero frequencies that experience no common shocks, and hence use the normalisation $N^{-1/2}$ to establish convergence of the aggregate to a stochastic process. Their model is formulated first in discrete time and then in continuous time as the sum of heterogeneous Ornstein–Uhlenbeck processes.

1.2. Incomplete information

Despite possessing many desirable empirical features, fractionally integrated models sometimes suffer from a serious shortcoming. As Granger (2000) p. 9 observes

> There is also little or no basic economic theory leading to fractional $I(d)$ variables, unlike the efficient market theory for $I(1)$.

This paper attempts to address this point, generating results where the micro-series are random walks, in line with 'the efficient market theory', but the macro-series will be $I(d)$. We examine the case when agents optimise under imperfect information, imagining they face shocks of differing durations that they are unable to tell apart. Agents use their own information efficiently: they base their actions on optimal forecasts, derived using techniques in Granger and Newbold (1977). Their own information is, however, incomplete in the sense that they do not know any other agent's shock. Models of incomplete information have a long history in economics for explaining behaviour both by firms and by households. As an example, we show that these conditions are created by a very plausible modification to the models of household consumption under uncertainty proposed by Goodfriend (1992) and Pischke (1995): heteroskedasticity of income shocks across the population. In the context of consumption, the capacity to generate long memory in an aggregate from a large number of $I(1)$ series means that aggregate consumption can be $I(d)$ even though every household obeys Hall's (1978) well-known random walk hypothesis.

In considering the aggregation, we adopt the general, semi-parametric, framework developed by Robinson (1978) and deployed in Lippi and Zaffaroni (1998) and Zaffaroni (2004), but we are forced to confront a number of issues not previously covered in this literature. The most important of these is that, because agents' behaviour is based on signal extraction between two types of shocks, their AR(1) parameter is a known function of the variances of those shock processes. This relationship gives rise to some new results with implications for modelling the aggregate. It becomes possible to generate orders of integration that are not bounded at one and idiosyncratic shocks that survive in the aggregate even after differencing.

This paper also sits within a wider literature linking the parameters of micro-series with those of the macro-series to which they aggregate, raising the possibility that information from one could enhance understanding of the other. Lewbel (1994) considers how the moments of the micro-distribution of a heterogeneous autoregressive parameter shape the autocorrelation function of the aggregate, without studying long memory directly. Abadir and Talmain (2002) explore the properties of aggregate output from a real business cycle model of heterogeneous firms operating under monopolistic competition. They examine the autocorrelation properties of GDP that results from the aggregation of non-linear relationships implied by the model when the firms receive temporary productivity shocks and show that the series exhibits a form of long memory, displaying more persistence than a standard ARMA model while also being mean reverting. As an example, our model is estimated on a panel study of US household consumption using maximum likelihood techniques that take full account of the linkages between parameters. Estimates of the order of integration are then made using kernel techniques to describe the behaviour of the population parameters.

Section 2 develops the basic model while Section 3 discusses how these same parameters would translate through to macroeconomic data. Both sections discuss household consumption as a particular example. To further this, the micro-model is estimated on a US household panel study in Section 4 and estimates of the key population parameters are made using kernel techniques. Extensions to the basic model are discussed in Section 5 and Section 6 concludes.

2. Economic underpinnings

2.1. Assumptions

We imagine an economy with a large number of agents, each of whom faces the same discrete-time individual long-run constrained optimisation problem of a kind widely used in economics. We adopt the straightforward conventions on behaviour and processes used widely in the literature on incomplete information. Assumption 1 ensures a closed form expression for each agent's behaviour under uncertainty; the remainder outline the information set. The assumptions are stated generally, the following discussion places them in a well-known context: household consumption.