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Nonparametric dynamic panel data models: Kernel estimation and specification testing

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ABSTRACT

Motivated by the first-differencing method for linear panel data models, we propose a class of iterative local polynomial estimators for nonparametric dynamic panel data models with or without exogenous regressors. The estimators utilize the additive structure of the first-differenced model—the fact that the two additive components have the same functional form, and the unknown function of interest is implicitly defined as a solution of a Fredholm integral equation of the second kind. We establish the uniform consistency and asymptotic normality of the estimators. We also propose a consistent test for the correct specification of linearity in typical dynamic panel data models based on the L_2 distance of our nonparametric estimates and the parametric estimates under the linear restriction. We derive the asymptotic distributions of the test statistic under the null hypothesis and a sequence of Pitman local alternatives, and prove its consistency against global alternatives. Simulations suggest that the proposed estimators and tests perform well for finite samples. We apply our new method to study the relationships among economic growth, the initial economic condition and capital accumulation, and find a significant nonlinear relation between economic growth and the initial economic condition.

1. Introduction

There exists an enormous literature on parametric (often linear) panel data models; see the books by Arellano (2003), Hsiao (2003), and Baltagi (2008) for an excellent overview. Nevertheless, the parametric functional forms may be misspecified, and estimators based on misspecified models are often inconsistent and thus invalidate subsequent statistical inferences. For this reason, we have also observed a rapid growth of the literature on nonparametric (NP) and semiparametric (SP) panel data models in the last two decades. See Su and Ullah (2011) for a recent survey on this topic.

To the best of our knowledge, there is a lack of satisfactory development in NP or SP dynamic panel data models where a lagged dependent variable enters the nonparametric component instead of a parametric (usually linear) component of the models. For NP panel data models with fixed effects, the main focus has been on the model

$$Y_{it} = m(X_{it}) + \alpha_i + \varepsilon_{it}, \tag{1.1}$$

$i = 1, \ldots, N$, $t = 1, \ldots, T$, where the functional form of $m(\cdot)$ is not specified, the covariate X_{it} is of $d \times 1$ dimensions, and α_i is a fixed effect that can be correlated with X_{it}, and the ε_{it}'s are idiosyncratic error terms. Motivated by the first-differencing method for linear panel data models, one can consider the following first-differenced model:

$$\Delta Y_{it} = m(X_{it}) - m\left(X_{i,t-1}\right) + \Delta\varepsilon_{it} \tag{1.2}$$

where $\Delta Y_{it} \equiv Y_{it} - Y_{i,t-1}$ and $\Delta\varepsilon_{it} \equiv \varepsilon_{it} - \varepsilon_{i,t-1}$. Li and Stengos (1996) suggest estimating $m(X_{it}, X_{i,t-1}) \equiv m(X_{it}) - m(X_{i,t-1})$ by first running a local linear regression of ΔY_{it} on X_{it} and $X_{i,t-1}$, and then obtaining estimates of $m(\cdot)$ by using standard methods for estimating nonparametric additive models, e.g., the marginal integration method of Linton and Nielsen (1995) or the backfitting method (see, e.g., Opsomer and Ruppert (1997) and Mammen et al. (1999)). Apparently, this method suffers from the "curse of dimensionality" problem because the first step of the local linear regression involves estimating a $2d$-dimensional nonparametric object, and it does not utilize the fact that the two additive components share the same functional form. In view of this, Baltagi

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and Li (2002a,b) obtain consistent estimators of $m(\cdot)$ by considering the first-differencing method and using series approximation for the nonparametric component.[1] Also on the basis of the first-differenced model in (1.2), Henderson et al. (2008) introduce an iterative nonparametric kernel estimator of $m(\cdot)$ and conjecture its asymptotic bias and variance and asymptotic normality. The crucial assumptions in this latter paper include: (1) the ε_{it}'s are independent and identically distributed (IID) across both i and t, and are independent of X_{it}, and (2) there exists a consistent initial estimator. In more recent work, Lee (2013) considers the sieve estimation of (1.1) when $X_{it} = Y_{i,t-1}$ via within-group transformation. That is, he considers the following transformed model:

$$Y_{it} - T^{-1}\sum_{s=1}^{T} Y_{is} = m\left(Y_{i,t-1}\right) - T^{-1}\sum_{s=1}^{T} m\left(Y_{i,s-1}\right) + \varepsilon_{it} - T^{-1}\sum_{s=1}^{T}\varepsilon_{is} \quad (1.3)$$

and approximates the unknown smooth function $m(\cdot)$ by some basis functions. Under the assumption that $\lim_{N,T\to\infty} N/T = c \in (0,\infty)$, he finds that the series estimator is asymptotically biased. So he proposes a bias-corrected series estimator and establishes its asymptotic normality.

Another method that is adopted to estimate the model in (1.1) is based on the profile likelihood or least squares method in the statistical literature. For example, Su and Ullah (2006a) propose estimating the unknown function by the profile least squares method under the identification condition $\sum_{i=1}^{N}\alpha_i = 0$, which boils down to a local linear analogue of the least squares dummy variable (LSDV) estimator for typical linear panel data models. Under the weaker identification condition that $E(\alpha_i) = 0$, Li and Sun (2011) propose mimicking the parametric LSDV estimation method by removing the fixed effects nonparametrically and establish the asymptotic normality of their estimator under the assumption that the ε_{it}'s are independent of α_j and $E(\varepsilon_{it}|\mathcal{X}) = 0$ for all i, j and t where $\mathcal{X} = \{X_{js}, j = 1,\ldots,N, s = 1,\ldots,T\}$, and that $T \to \infty$ sufficiently fast as $N \to \infty$.

In addition, it is worth mentioning that there are also some studies on semiparametric panel data models that include the model in (1.1) as a special case. One example is the paper by Sun et al. (2009) who consider the local linear estimation of the varying coefficient panel data models with fixed effects

$$Y_{it} = Z_{it}'\theta\left(X_{it}\right) + \alpha_i + \varepsilon_{it}, \quad i = 1,\ldots,N, t = 1,\ldots,T, \quad (1.4)$$

where the idiosyncratic error terms ε_{it} are independent of X_{js}, Z_{js} and α_j for all i, j, t, and s. Note that this model reduces to (1.1) when $Z_{it} \equiv 1$. Obviously, they do not allow $Y_{i,t-1}$ to enter X_{it}. Another example is the study of the partially linear panel data models with fixed effects: Baltagi and Li (2002a,b) propose a semiparametric instrumental variable estimator for estimating a partially linear dynamic panel data model; Qian and Wang (2012) consider the marginal integration estimator of the nonparametric additive component resulting from the first-differencing of a partially linear panel data model. Unfortunately, none of these models allow the lagged dependent variable to enter the nonparametric component. In addition, Mammen et al. (2009) consider the consistent estimation of nonparametric additive panel data models with time effects or with both time and individual effects via backfitting. But they only establish the asymptotic normality of the resulting estimator in the presence of time effects only.

[1] Both Li and Stengos (1996) and Baltagi and Li (2002a,b) consider a more general model, namely, a partially linear model.

In this paper, we propose an iterative kernel estimation of nonparametric dynamic models of the form

$$Y_{it} = m\left(Y_{i,t-1}, X_{it}\right) + \alpha_i + \varepsilon_{it}, \quad i = 1,\ldots,N, t = 1,\ldots,T, \quad (1.5)$$

where X_{it} is a $d \times 1$ vector of regressors, $m(\cdot,\cdot)$ is an unknown but smooth function defined on $\mathbb{R}^{d+1}$, the α_i's are the individual-specific fixed effects, and the ε_{it}'s are idiosyncratic error terms. Let $\underline{X}_{it} \equiv \left(X_{it}',\ldots,X_{i1}'\right)'$ and $\underline{Y}_{i,t-1} \equiv \left(Y_{i,t-1}',\ldots,Y_{i1}'\right)'$. We assume that $E\left(\varepsilon_{it}|\underline{Y}_{i,t-1},\underline{X}_{it}\right) = 0$ and consider the first-differenced model

$$\Delta Y_{it} = m\left(Y_{i,t-1}, X_{it}\right) - m\left(Y_{i,t-2}, X_{i,t-1}\right) + \Delta\varepsilon_{it}. \quad (1.6)$$

Apparently, (1.6) has a simple additive structure. But it is different from the typical additive models in two aspects. First, the error term $\Delta\varepsilon_{it}$ forms a moving averaging process of order 1 (MA(1)) and is correlated with the regressor $Y_{i,t-1}$ in general. Second, the two additive components in (1.6) share the same functional form. The first observation indicates that the traditional kernel estimation based on either marginal integration or the backfitting method does not yield a consistent estimator of $m(\cdot,\cdot)$. The second observation, in conjunction with the fact that $E\left[\Delta\varepsilon_{it}|Y_{i,t-2}, X_{i,t-1}\right] = 0$, implies that $m(\cdot,\cdot)$ implicitly solves a Fredholm integral equation of the second kind (see (2.7) in Section 2.1), so we can propose a simple local polynomial regression-based iterative estimator for it.

Under fairly general conditions which allow nonstationarity of $(Y_{i,t-1}, X_{it}, \varepsilon_{it})$ along the time dimension and conditional heteroskedasticity along ε_{it}, we establish the uniform consistency of the proposed estimator over a compact set and study its asymptotic normality by passing the cross-sectional unit N to ∞ and holding the time dimension T as a fixed constant as in typical micropanel data models. We also remark that under some suitable conditions, one can plug our estimate of $m\left(Y_{i,t-1}, X_{it}\right)$ into (1.6) to obtain a new estimate of $m(\cdot)$ to achieve certain "oracle" properties.

On the basis of our kernel estimator, we also propose a test for the correct specification of linear dynamic panel data models. There have been various specification tests for parametric panel data models in the literature; see Hausman (1978), Hausman and Taylor (1981), Arellano (1990), Arellano and Bond (1991), Li and Stengos (1992), Metcalf (1996), Baltagi (1999), Fu et al. (2002), Inoue and Solon (2006) and Okui (2009), among others. Nevertheless, none of these tests are designed to check the correct specification of linearity in the panel data models. In more recent work, Lee (2011) proposes a class of residual-based specification tests for linearity in dynamic panel data models by characterizing the correct specification of linearity as the martingale difference property of the error terms in the model and extending the generalized spectral analysis of Hong (1999) to dynamic panel data models. To eliminate the problem of incidental parameters, she focuses on dynamic panel data models with both large N and large T. So her test cannot be applied to typical micropanel data where T is usually small.

In this paper, we consider a specification test for the linearity of a dynamic panel data model when N is large and T is small/fixed. Under the null hypothesis of correct specification of linear dynamic panel data models, various methods can be called upon for estimating the unknown parameters in the linear regression model. Under the alternative, the functional form of the regression model is left unspecified as in (1.5) and one can estimate the unknown function by using the method proposed in this paper. We base our test statistic on the L_2 distance between the two functional estimates in the spirit of Härdle and Mammen (1993), and study its asymptotic properties under the null hypothesis, a sequence of Pitman local alternatives, and global alternatives.