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Energy flow analysis of mid-frequency vibration of coupled plate structures with a hybrid analytical wave and finite element model

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ABSTRACT

The medium frequency vibration of a built-up plate structure is studied by an energy flow analysis which extends the concept of statistical energy analysis. The propagative waves of the plates are considered as subsystems that carry and spread energy. Symplectic analytical solutions for mode count, modal density and group velocity of each wave subsystem are obtained based on accurate consideration of the plate geometry and boundary conditions, while the joint vibrational behavior is described by a finite element model. The input mobility and coupling factor associated with each wave subsystem are accurately obtained using a hybrid analytical wave and finite element formulation. Based on the power balance relation of each wave subsystem, the system energy equations are established. Numerical examples for built-up structures comprising rectangular plates demonstrate high accuracy and efficiency. In contrast with statistical energy analysis, the energy of each wave subsystem can be obtained, facilitating the understanding and control of structural vibration and local response. The computational time of the hybrid formulation decreases significantly with increasing length/width ratio of the plates. The wave scattering property of the joint can also be obtained and used to replace the finite element model in repetitive analysis.

1. Introduction

In the design of engineering structures, the analysis of vibration and acoustic behavior is of great importance for the evaluation of energy consumption, noise, comfort, safety and fatigue life. In vibro-acoustics, the audio frequency range is conventionally divided into low, medium and high frequency regions. At low frequencies, the structural response shows obvious peak values, and the influence of geometric shape and boundary conditions on the results can be clearly observed. In this range, traditional deterministic methods such as the finite element method (FEM) [1] and the boundary element method (BEM) [2] are appropriate. At high frequencies, smooth response behavior due to modal overlap can be observed and is significantly affected by structural uncertainties. Statistical energy analysis (SEA) [3] is most frequently used in this range. For mid-frequency vibration analysis, the traditional deterministic methods suffer disadvantages of huge computational load and low accuracy, while the statistical methods suffer the limitation that the structural uncertainty is insufficient.

At present, there are three main approaches to the analysis of medium frequency vibration [4]. The first approach is to develop methods with higher efficiency based on the standard FEM or BEM [5–10], or on wave theory [4,11–17], i.e. extending the range of low frequency deterministic analysis. The second approach is to develop methods based on SEA with more relaxed assumptions [18–21], i.e. extending the range of high frequency statistical analysis. The third approach is to analyze structures by a hybrid framework that combines deterministic methods and statistical methods [22–26].

As an energy flow method, SEA has a great advantage of computational efficiency compared with the displacement based methods when calculating the energy response of structures. SEA is said to be an ad hoc extension of the exact results which may be derived for two coupled single degree of freedom oscillators under broadband excitation [27], and also based on many assumptions of high frequency [28]. At medium frequencies, the geometric shape and boundary conditions significantly affect the dynamics of the structure, violating the basic assumption of SEA and the classical

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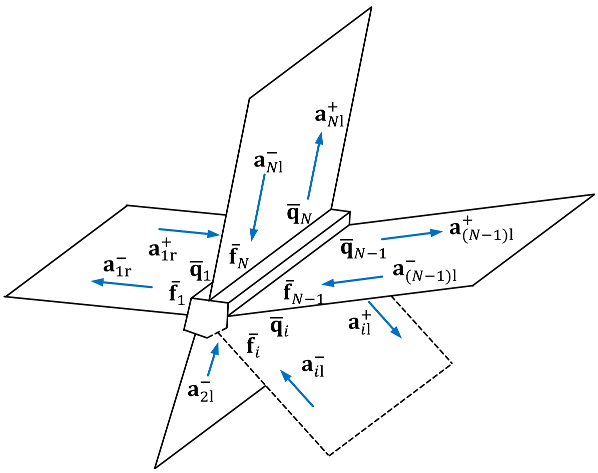

Fig. 1. Schematic of coupled plate structure and wave scattering at the joint, where $\mathbf{a}_{i\mathrm{l}}^{+}$, $\mathbf{a}_{i\mathrm{l}}^{-}$, $\mathbf{a}_{i\mathrm{r}}^{+}$, and $\mathbf{a}_{i\mathrm{r}}^{-}$ are positively and negatively travelling waves at the left and right ends of plate i, $\bar{\mathbf{q}}_i$ and $\bar{\mathbf{f}}_i$ are nodal displacement and internal force vectors at the coupling interface of plate i $(i = 1, \ldots, N)$.

asymptotic formulae of its parameters, e.g. modal density, coupling loss factor and input power. Even at high frequencies, the classical formulae of SEA parameters are not appropriate for all structural forms. Therefore, much effort has been made to find more appropriate SEA parameters. Xie et al. [29] investigated the effects of boundary conditions on the mode count and modal density of beams and plates using the wavenumber integration method. Seçgin [30] determined SEA parameters for point connected, directly coupled symmetrically laminated composite plates using a numerical modal based approach. Finnveden [31] calculated the modal density, group velocity and coupling loss factor of a waveguide structure using the waveguide FEM. Based on a dual modal formulation and the power injection method, respectively, Totaro et al. [32], Bies and Hamid [33] calculated the coupling loss factor used in SEA. Langley and Heron [34] calculated the transmission coefficients and the associated coupling loss factors for a semi-infinite built-up plate structure using the wave dynamic stiffness method. Using FEM to calculate SEA parameters has advantages in handing complex geometry and boundary constraints [35–38].

In this paper, a new energy flow method is proposed for the analysis of mid-frequency vibration of built-up plate structures based on the power balance framework used in SEA. The established concept of using the parameters of input mobility, damping loss factor and coupling loss factor to describe the input, transmission and dissipation of the energy of each subsystem in the classical SEA is adopted in the presented energy flow method. In contrast with classical SEA, in which the whole flexural or in-plane wave field of the plates is regarded as a subsystem, each pair of propagative waves of the plates is considered as a subsystem in this paper. Based on this wave subsystem description, the analytical expressions of the SEA parameters can be obtained. Similar ideas have been used in previous studies. For example, the wave intensity analysis proposed by Langley [18] considers the waves in each direction as subsystems based on which the system power balance equations can be established. Based on [39], Wester and Mace [40] adopted analytical waves as subsystems in the SEA of two directly coupled rectangular plates. However, the analytical wave components can only be obtained for plates with two opposite edges simply supported. To overcome this boundary condition limitation the wave FEM [13,14] or the waveguide FEM [31] may be considered. However, because a finite element model is introduced, the advantages in accuracy, efficiency and parametric analysis of the analytical wave description are lost. In this paper, the boundary condition limitation of the traditional analytical wave description is avoided by using the symplectic method [41–45] to describe the vibration behavior of the plates. And then the parameters of input mobility, damping loss factor and coupling loss factor can be obtained using the symplectic analytical waves which exactly reflect the mid-frequency characteristic of the built-up structure. Based on the symplectic analytical wave modes (i.e. wave propagation parameters and wave shapes) of rectangular plates previously obtained by the authors [43,44], the mode count, modal density and group velocity of each wave subsystem are obtained in symplectic analytical form. Like the coupling loss factor used in the classical SEA, the power transmission between the wave subsystems is described by a coupling factor which is finally transferred into the solution of the scattering coefficients associated with the waves of each plate. A hybrid analytical wave and finite element formulation is established by combining the analytical wave description for the plate components and the finite element description for the joint to obtain the input mobility and the scattering property of each wave subsystem with high accuracy and efficiency. Hence, the parameters of the energy flow analysis can be obtained exactly and reflect the mid-frequency characteristics of the structure. Then by considering the power balance the system equations of the energy analysis of the built-up structure are established. Solving the system equations of energy equilibrium directly gives the energy of each wave subsystem. Therefore, compared with SEA, from which only the energy of the whole flexural or in-plane wave field of plates can be obtained, the energy flow analysis presented in this paper can provide more insight into the vibration transmission, and therefore more useful guidance for the control of vibration of the built-up structure.

The paper is organized as follows. Following this introduction, a wave subsystem based description of the transmission of the structural vibration is presented. Then the parameters for the energy flow analysis are derived and the system equations of power balance of the built-up structure are established. Next, the hybrid analytical wave and finite element solution formulation is derived. Finally, the effectiveness of the methods presented is validated based on three kinds of structural forms including a single rectangular plate, two co-planar plates coupled via a structural joint and three angle coupled plates with a structural joint.

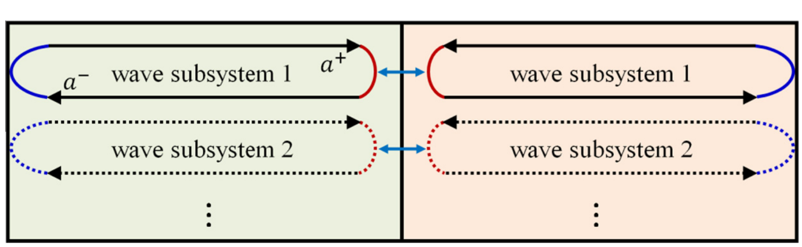

Fig. 2. Schematic of energy transmission between two plates based on propagative wave subsystem description.