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# A new hybrid uncertain analysis method for structural-acoustic systems with random and interval parameters

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ABSTRACT

This paper proposes an uncertain analysis method named Polynomial Chaos Response Surface method (PCRSM) for uncertainty propagation in structural-acoustic system containing hybrid uncertainties. The polynomial chaos expansion method combined with the response surface methodology is employed to handle the random and interval uncertainty. The PCRSM does not require efforts to modify model equations due to its non-intrusive characteristic. The PCRSM is also able to predict the response of the system with only random or only interval uncertainty. The numerical results demonstrate that the PCRSM is effective and computationally efficient.



## 1. Introduction

With the increasing demands on sound quality, the noise control of the enclosed cavity such as the car passenger compartments and the airplane cabins is becoming a challenging problem encountered by many engineers. It has been found that the noise in the enclosed cavity is significantly affected by the vibrating structure, acoustic cavity and coupled solid-fluid interface [1]. Traditional numerical methods for the response analysis of the structural-acoustic system are conducted under the assumption that the physical properties and boundary conditions are deterministic. However, a complicated structural-acoustic system often contains many uncertain factors, such as the model inaccuracies, the unpredictable loading conditions, and the aggressive environment factors. Hence, the response of the structural-acoustic system is always subjected to these uncertain parameters. The most common approaches to model these uncertainties are the probabilistic methods and the non-probabilistic methods. The probabilistic methods have been used to deal with the uncertain problems with sufficient information, and the uncertain parameters are modeled as random variables through the description of the predefined probability distribution functions. Among these probabilistic methods, the Monte Carlo method [2,3] is widely used due to the ease of implementation and its robust characteristic. The main shortcoming of the MCM is its large computational requirements especially for time-consuming problems. Thus, the MCM is usually used as an approach for validating the accuracy and efficiency of other methods. The perturbation method [4–6] is an alternative approach for the probabilistic problems. In this method, both input and output parameters are expanded using Taylor series with random variables. Therefore, the perturbation method is efficient only for small uncertainty level of parameters and seems not suitable to solve a problem of dynamics for frequencies near resonance. Spectral stochastic method [7], an efficient alternative for stochastic problems, is based on a series expansion to model the relationship between input and output quantities. Compared with the solution obtained by MCM, the polynomial chaos expansion method, one of the spectral stochastic method, shows better efficiency and reasonable accuracy. There are two main polynomial chaos expansion approaches, categorized as intrusive and non-intrusive. The intrusive approach involves the substitution of all uncertain variables and parameters in the governing equations with the polynomial expansions consisting of unknown polynomial coefficients. However, modifications on the governing equations of the system may be complex or even impossible if the governing equations describing the physical model are not available. Non-intrusive polynomial chaos expansion techniques were developed to overcome this limitation. The polynomial chaos coefficients may be efficiently computed by means of the non-intrusive techniques such as the projection method [8,9] or the regression method [10,11]. The non-intrusive polynomial chaos methods have been applied in uncertainty propagation in various fields, such as

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computational fluid dynamics [12], acoustic fields [13], structural dynamics [14], and structural-acoustic field [15] and so on. Based on the generalized polynomial chaos expansion technique, Saha et al. [14] introduced a non-intrusive stochastic simulation method for the dynamic response analysis of the base-isolated liquid-storage tanks under considering parametric uncertainties. Sepahvand et al. [15] adopted the non-intrusive polynomial chaos expansion method to investigate the vibroacoustic performance of laminated composite plates with uncertain elastic parameters.

The probabilistic approaches give reliable results only when sufficient statistical data are available to obtain the probability distribution of uncertain parameters. In order to handle the uncertainties without sufficient information, the non-probabilistic methods can be employed. Interval analysis approach [16], one of the non-probabilistic methods, has achieved widespread attention. The upper and lower bounds of the response can be calculated directly based on the interval mathematics, with only knowing the bounds of the uncertain parameters. However, the main shortcoming of the interval analysis approach is the overestimation caused by the wrapping effect [17–19]. Many efforts have been made to reduce overestimation, and the related articles can be found in [20–22]. Muhanna et al. [20] developed a new interval approach based on an element by element (EBE) technique for the treatment of parameter uncertainty for linear static problems of mechanics. In the literature [21], a modified interval perturbation finite element method was proposed for the frequency response analysis of the structural-acoustic system with interval parameters. Xia et al. [22] exploited the subinterval perturbation finite element method to predict the frequency response of the structural-acoustic system with large uncertain-but-bounded parameters. The numerical results of these methods above have verified the effectiveness on reducing the overestimation. However, these methods require more efforts on modifying the governing equations of the system, which are often complicated in practical engineering problems. Recently, Wu et al. [23] proposed a new non-intrusive interval method for the dynamic response of nonlinear systems with interval parameters using Chebyshev polynomial series. Fang et al. [24] developed a new interval response surface model for the purpose of efficiently implementing the interval model updating procedure. In this method, the overestimation due to the use of interval arithmetic can be maximally avoided leading to accurate estimation of the response intervals. By combining the MCM and the response surface methodology, Zou et al. [25] proposed a method to study the uncertainty propagation in accident reconstruction model with interval parameters.

As aforementioned, it can be seen that most researches on uncertain problems have been carried out by using either interval model or random model. However, the random and interval parameters may exist simultaneously in the real uncertain engineering problems. There are some researches on the hybrid uncertainty method which has been widely applied to the structural reliability analysis [26,27] and prediction of structural response [28–31]. Wang and Qiu [26] proposed a reliability-based design method for static analysis of structures with both random and interval analsyis. Gao et al. [28] proposed a mixed perturbation Monte-Carlo method (HPMCM) to predict the intervals of the expectation and variance of the structural responses. Based on the vertex method and the random interval moment method, Xia et al. [29] developed a hybrid perturbation vertex method (HPVM) for the frequency response analysis of structural-acoustic field with a mixture of random and interval parameters. The HPVM shows its superiority in the aspect of computational efforts, but the simulation results of HPVM cannot be accurate when the extreme values of the response are not at the vertexes. Most of these hybrid uncertain methods are mainly based on perturbation theory.

From the overall perspective, research on the hybrid uncertain problems is still in its preliminary stage and some important issues still remain unsolved. First, the present methods for hybrid uncertain problems are mainly based on perturbation theory, which is not suitable for the case that the governing equations of the problem are complex or inaccessible. An efficient and effective non-intrusive method to evaluate the intervals of expectation and standard variance of the response vector of the hybrid uncertain problems has still not been developed. Second, the application of non-intrusive method for the hybrid uncertain problems, especially for the frequency response analysis of structural-acoustic systems, has hardly been exploited yet.

In this paper, our efforts are concentrated specifically on developing a new uncertain analysis method, which is designed with ease of use and non-intrusiveness in mind, for the frequency response prediction of structural-acoustic system with random and interval parameters. Because of manufacturing and measurement errors, the parameters of the structure such as mass density and Young's modulus, whose probability distributions can be obtained from sufficient information, are regarded as random variables. Considering the unpredictable environment temperature, it is practically hard to obtain the probability distributions of some parameters such as density of air and acoustic speed of air surrounding the acoustic cavity, but their variation ranges are limited inside some intervals. Therefore, these parameters whose lower and upper bounds can be determined by the limited information are treated as interval variables. In this work, we employ a non-intrusive polynomial chaos expansion method to deal with the random variables, because a particularly attractive feature of the polynomial chaos expansion method is the easiness to recover information and characterize the variability, in particular the mean and the variance. As the structural-acoustic system includes both the random and interval variables, the coefficients obtained from non-intrusive polynomial chaos expansion method are not determined values but depend on interval variables. Recently, response surface methodology combined with interval analysis has demonstrated its ability to tackle the engineering problems with interval parameters [16,17,24,25]. Then we use the response surface method to handle the interval parameters due to its simplicity and ease of use. Finally, the upper and lower bounds of the probabilistic characteristics can be efficiently computed by MCM.

The manuscript is structured as follows. In Section 2, the dynamic equation of the structural-acoustic systems with random and interval parameters is derived. In Section 3, the proposed PCRSM for frequency response analysis of the structural-acoustic systems with random and interval parameters is presented. In Section 4, three structural-acoustic models with random and interval parameters are conducted to demonstrate the effectiveness of the proposed methods. Several conclusions drawn based on the current study are given in Section 5.

## 2. Structural-acoustic FE model with random and interval parameters

The dynamic equation of the structural-acoustic system is

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{C}\dot{\mathbf{U}} + \mathbf{K}\mathbf{U} = \mathbf{f} \tag{1}$$

where

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_S & 0 \\ \rho_0 c_0^2 \mathbf{H}_{SF}^T & \mathbf{M}_F \end{bmatrix} \quad \mathbf{K} = \begin{bmatrix} \mathbf{K}_S & -\mathbf{H}_{SF} \\ 0 & \mathbf{K}_F \end{bmatrix}$$
$$\mathbf{U} = \begin{bmatrix} \mathbf{d}_S \\ \mathbf{p}_F \end{bmatrix} \quad \mathbf{f} = \begin{bmatrix} \mathbf{f}_{b,S} \\ \mathbf{f}_{b,F} \end{bmatrix} \tag{2}$$

$\mathbf{M}_S$, $\mathbf{K}_S$, $\mathbf{d}_S$, $\mathbf{f}_{b,S}$ represent the structural mass matrix, stiffness matrix, nodal displacement vector, external body load applied to the