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Efficient reliability assessment of structural dynamic systems with unequal weighted quasi-Monte Carlo Simulation

Jun Xu [a,b,*], Wangxi Zhang [a], Rui Sun [a]

[a] *Department of Structural Engineering, College of Civil Engineering, Hunan University, Changsha 410082, PR China*
[b] *Hunan Provincial Key Lab on Damage Diagnosis for Engineering Structures, Hunan University, Changsha 410082, PR China*

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ABSTRACT

An efficient method for reliability assessment of structural dynamic systems is proposed. In the proposed method, the reliability is evaluated by the equivalent extreme value distribution of structural dynamic systems, where an estimator-corrector scheme based on the principle of maximum entropy with fractional moments as constraints is adopted to derive such distribution. The fractional moments, which are of paramount importance to the efficiency and accuracy of reliability assessment, can be obtained by a multidimensional integral. To calculate such an integral, a new approach named unequal weighted quasi-Monte Carlo Simulation is put forward. In this approach, different strategies for the construction of unequal positive weights, the rearrangement of sampling points to be integration points and the determination of required number of points are explored. The integral for the evaluation of fractional moments is then determined efficiently based on the obtained integration points and the weights straightforwardly. Numerical example is studied to verify the proposed method. The investigations indicate that the proposed method is of accuracy and efficiency for reliability assessment of structural dynamic systems, with only a few hundreds of deterministic dynamic response analysis for a practical problem involving multiple random parameters.

1. Introduction

Engineering structures are often subject to dynamic excitations, such as wind, sea waves and seismic motions, which inherently possess random characteristics [32]. On the other hand, the parameters of the structure, such as the material properties, the geometrical parameters and the boundary conditions, usually cannot be determined exactly in practical applications [24]. As a result, the randomness in both excitations and structural parameters may lead to considerable fluctuation in structural behaviors. Therefore, the reliability assessment of structural dynamic systems considering randomness is of paramount importance for providing a quantitative basis to ensure the structural safety [25].

Performing a reliability based analysis of structural dynamic systems is closely related to the determination of the probability that the structural dynamic response crosses a prescribed threshold for the first time over a given time interval, which is known as the first passage problem [29,34,22]. For the evaluation of the first passage probability, analytical methods, such as the Rice's formula [29], can only be used in very special cases and are hence not applicable to general engineering problems [17]. Alternatively, approximate methods are developed, i.e. Kolmogorov equation [36], however, the practical applicability may need prohibitive computational efforts in the case of complex multiple-degree-of-freedom (MDOF) structures.

Further, the extreme value distribution (EVD) is intimately related to the first passage reliability evaluation [7]. Likewise, analytical approaches, e.g. the formal series solution by Rice [18], and approximate approaches such as taking the Rayleigh distribution as the EVD [28] are investigated. Nevertheless, these results are actually achieved for some particular stochastic dynamic responses [15]. For a general stochastic process, how to obtain the EVD is still a difficult problem. Fortunately, the equivalent extreme value event has been developed recently by Li et al. [26], in which the correlation information of stochastic dynamic response is inherent. Through this, the EVD can be formulated and therefore the reliability can be assessed accordingly.

In this paper, an efficient method is proposed to derive the EVD for reliability assessment of structural dynamic systems with

* Corresponding author at: Department of Structural Engineering, College of Civil Engineering, Hunan University, Changsha 410082, PR China.

E-mail addresses: xujun86@hnu.edu.cn (J. Xu), wxizhang2000@163.com (W. Zhang).

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accuracy. In this regard, in Section 2, an estimator-corrector scheme is developed to capture the EVD by the principle of maximum entropy, in which the evaluation of fractional moments is involved. Next, in Section 3, a new approach named unequal weighted quasi-Monte Carlo Simulation (MCS) is proposed to efficiently obtain the required fractional moments without loss of accuracy. In such an approach, the construction of positive unequal weights, the rearrangement of sampling points to be integration points and the determination of required number of integration points are of great concern, where different strategies are developed accordingly. In Section 4, numerical example is illustrated to elucidate the proposed method. Concluding remarks are contained in the final section.

2. Reliability assessment of structural dynamic systems

2.1. The equivalent extreme-value event

The equivalent extreme value event consists of the following three theorems [26]:

Theorem 1. *Suppose $U_1, U_2, \ldots, U_m$ are m random variables. Let $W_{min} = min_{1\leqslant j\leqslant m}(U_j)$, then it goes that*

$$Pr\left\{\bigcap_{j=1}^{m}(U_j > a)\right\} = Pr\{W_{min} > a\} \tag{1}$$

where Pr denotes the probability for short, a is the threshold.

Theorem 2. *Suppose $U_1, U_2, \ldots, U_m$ are m random variables. Let $W_{max} = max_{1\leqslant j\leqslant m}(U_j)$, then it goes that*

$$Pr\left\{\bigcup_{j=1}^{m}U_j > a\right\} = Pr\{W_{max} > a\} \tag{2}$$

Theorem 3. *Suppose $U_{ij},\ i = 1, 2, \ldots, n;\ j = 1, 2, \ldots, m$ are $m \times n$ random variables. Let $W_{ext} = max_{1\leqslant i\leqslant n}(min_{1\leqslant j\leqslant m}(U_{ij}))$, then it goes that*

$$Pr\left\{\bigcup_{i=1}^{n}\left(\bigcap_{j=1}^{m}U_j > a\right)\right\} = Pr\{W_{ext} > a\} \tag{3}$$

In Eqs. (1)–(3), $W_{\min}, W_{\max}$ and W_{ext} are called the equivalent extreme value random variables and the corresponding random event is called the equivalent extreme value event, i.e. $W_{\min} > a$. These theorems have been proved rigorously in Ref. [26]. It is also worth pointing out that the correlation information of original random variables is retained in the equivalent extreme value event.

2.2. Reliability assessment of structural dynamic systems

Generally, it is easy to write the reliability of structural dynamic systems in the form of limit state function

$$R = \Pr\{G(t) > 0, \quad t \in [0, T]\} \tag{4}$$

where $G(t)$ is a time-dependent limit state function, T is the given time duration.

In fact, $G(t)$ is a stochastic process, of which the value at each time step can be regarded as a random variable. Therefore, Eq. (4) can be written equivalently in another form such that

$$R = \Pr\left\{\bigcap_{t\in[0,T]} G(t) > 0\right\} \tag{5}$$

According to Theorem 1, the equivalent extreme value can be expressed as

$$W_{\min} = \min_{t\in[0,T]}(G(t)) \tag{6}$$

The reliability in Eq. (5) can be rewritten as

$$R = \Pr\{W_{\min} > 0\} \tag{7}$$

Further, Eq. (7) can be changed as

$$R = \int_0^{\infty} p_{W_{\min}}(w)dw \tag{8}$$

where $p_{W_{\min}}(w)$ is the EVD.

Then, the first-passage reliability problem is transformed to be such a simple one-dimensional integration.

Once the system reliability is interested, which means multiple limit state functions are involved, there exists

$$R = \Pr\left\{\bigcap_{j=1}^{m}(G(t) > 0, \quad t \in [0, T_j])\right\} \tag{9}$$

where m is the number of limit state functions and T_j is the time duration for $G_j(t)$.

The equivalent extreme value can be defined as

$$W_{\text{ext}} = \min_{1\leqslant j\leqslant m}\left(\min_{t\in[0,T_j]}(G_j(t))\right) \tag{10}$$

Thus, the reliability can be determined by

$$R = \Pr\{W_{\text{ext}} > 0\} = \int_0^{\infty} p_{W_{\text{ext}}}(w)dw \tag{11}$$

where $p_{W_{\text{ext}}}(w)$ is the EVD for system reliability assessment.

As is seen, the EVD is of paramount importance for reliability assessment of structural dynamic systems. To this end, the principle of maximum entropy with fractional moments will be employed to derive such EVD.

2.3. The principle of maximum entropy based derivation of the extreme-value distribution with fractional moments

For simplicity, denote the EVD $p_{W_{\min}}(w)$ or $p_{W_{\text{ext}}}(w)$ as $p_Z(z)$. According to the information theory, the entropy of a continuous random variable with distribution $p_z(z)$ is defined as [21]

$$\underline{\mathbf{H}}[p_z(z)] = -\int_Z p_Z(z)\log[p_Z(z)]dz \tag{12}$$

A solution of the unknown $p_Z(z)$ can be obtained by using the principle of maximum entropy, in which the fractional moments are considered as the constraints. Specifically, we have to solve the constrained non-linear optimization problem:

$$\begin{cases} Maximize: \ \underline{\mathbf{H}}[p_Z(z)] = -\int_Z \log[p_Z(z)]dz \\ s.t. \ \begin{cases} \int_{-\infty}^{+\infty} p_Z(z)dz = 1 \\ \mu^{\alpha_k} = \int_Z z^{\alpha_k} p_Z(z)dz \end{cases} \text{for} \quad k = 1, 2, \ldots, K \end{cases} \tag{13}$$

where μ^{α_k} is an α_kth order fractional moment, K is the total number of fractional moments constraints. As is known from the probability theory, the probability density function is determined once all the integer order moments are known if they are finite [38]. The reason of using fractional moments as constraints is that an αth order fractional moment contains information about a large number of integer order moments [39]. Fractional moments begin to be employed to characterize a random variable from the pioneering work in Refs. [9,10,12]. If integer moments are applied, a large number of moments are required to achieve a reasonable accuracy in the modeling of the distribution tail, where the entropy optimization algorithm may become numerical instability as the number of moment constraints becomes large [40].