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Testing for a break in trend when the order of integration is unknown[☆]

Fabrizio Iacone[a], Stephen J. Leybourne[b], A.M. Robert Taylor[c,*]

[a] *Department of Economics and Related Studies, University of York, UK*

[b] *School of Economics and Granger Centre for Time Series Econometrics, University of Nottingham, UK*

[c] *Essex Business School, University of Essex, UK*

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ABSTRACT

Harvey, Leybourne and Taylor [Harvey, D.I., Leybourne, S.J., Taylor, A.M.R. 2009. Simple, robust and powerful tests of the breaking trend hypothesis. Econometric Theory 25, 995–1029] develop a test for the presence of a broken linear trend at an unknown point in the sample whose size is asymptotically robust as to whether the (unknown) order of integration of the data is either zero or one. This test is not size controlled, however, when this order assumes fractional values; its asymptotic size can be either zero or one in such cases. In this paper we suggest a new test, based on a sup-Wald statistic, which is asymptotically size-robust across fractional values of the order of integration (including zero or one). We examine the asymptotic power of the test under a local trend break alternative. The finite sample properties of the test are also investigated.

1. Introduction

In this paper we focus on the issue of testing for a break, at some unknown point, in the deterministic linear trend component of a time series whose stochastic component is a fractionally integrated process of order δ, $I(\delta)$. It is important to be able to detect a break in the deterministic trend function to avoid the detrimental effect of its neglect on subsequent inference on, for example, the order of integration of the data; see, *inter alia*, Perron (1989) and Busetti and Harvey (2001) where the inference concerns the $I(0)/I(1)$ dichotomy.

The problem is that the inference arising from tests for the existence of a trend break is itself typically contingent on the order of integration δ of the data, which is, of course, unknown in practice. For instance, Chu and White (1992) develop trend break tests based on sup-Wald and CUSUM statistics under the assumption that $\delta = 0$. Wright (1998), however, shows (in the context of testing for a break in the level) that tests of this kind spuriously reject in favour of a break when $\delta > 0$. In the case where δ can only assume the values zero or one (and it is not known which is true), Harvey et al. (2009) [HLT] propose a GLS-based trend break test that is (asymptotically) size-robust under either regime. However, this test is not size controlled for any other values of δ; its asymptotic size can be one, when, for example, $\delta \in (0, 1/2)$, or zero, when $\delta \in (1/2, 1)$. The former property falsely implies the presence of a broken trend in the data, which hinders any subsequent modelling/inference efforts undertaken due to a loss of efficiency in parameter estimation. The latter property has potentially more serious consequences since it can result in the neglect of a (true) broken trend term. In turn, this leads to inconsistent model parameter estimation—including estimation of the memory parameter δ. Obviously this would have profoundly negative implications on the size, or power, of any conducted hypothesis tests concerning δ.

Both these factors considerably limit the appeal of the HLT approach, given the recent level of interest in employing fractionally integrated time series models to characterise economic and financial data. With this in mind, our aim in this paper is to provide a trend break test which is (asymptotically) size-robust but which, unlike the HLT test, remains valid without placing unreasonable constraints on the allowable values of δ.

Our approach follows the GLS spirit of HLT, based on constructing a trend break test statistic after taking δ-differences of the data—the difference being that we do not constrain δ to be zero or one. Since we do not assume knowledge of the break location, following Andrews (1993), we then suggest a sup-Wald statistic which tests for the trend break over all possible candidate dates. To render our statistic feasible, we require an estimate of δ, $\hat{\delta}$, such

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* Correspondence to: Essex Business School, University of Essex, Wivenhoe Park, Colchester, Essex, CO4 3SQ, UK. Tel.: +44 0 115 846 8385.

E-mail address: Robert.Taylor.1992@pem.cam.ac.uk (A.M. Robert Taylor).

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that the statistic is based on $\hat{\delta}$-differences of the data. For $\hat{\delta}$ we employ the Fully Extended Local Whittle [FELW] estimator of Abadir et al. (2007) [ADG], which is constructed under the null of no trend break. We show that this estimator has the necessary consistency properties for δ, even in the presence of a local break in trend. Our test procedure is shown to have a limit distribution under the no break null hypothesis which still depends on δ but using $\hat{\delta}$ we are able to provide a test which is asymptotically size controlled irrespective of the value of δ (within a prescribed range).

The remainder of the paper is organised as follows. Section 2 introduces the fractionally integrated trend break model and describes our proposed test statistic. Section 3 establishes its large sample properties in a local trend break setting and provides asymptotic null critical values for the test. Issues relating to the practical application of our test, are discussed in Section 4. In Sections 5 and 6 we present an evaluation of the finite sample size and power properties, respectively. Section 7 concludes the paper.

In what follows we use the following notation: '$x := y$' ('$x =: y$') to indicate that x is defined by y (y is defined by x); '$\overset{d}{=}$' to denote equivalence in distribution; $\lfloor\cdot\rfloor$ to denote the integer part of the argument; '$\overset{p}{\rightarrow}$' to denote convergence in probability and '$\overset{d}{\rightarrow}$' to denote convergence in the Skorohod J_1 topology of $D[0, 1]$, the space of real-valued functions on [0, 1] which are continuous on the right and with finite left limit, respectively, as the sample size diverges to positive infinity, and $\mathbb{I}(\cdot)$ to denote the indicator function.

2. The fractionally integrated trend break model and trend break test

We consider the following trend break data generation process (DGP)

$$y_t = \beta_1 + \beta_2 t + \beta_3 DT_t(\tau_0) + u_t, \quad t = 1, \ldots, T, \tag{2.1}$$

with $DT_t(\tau_0) := (t - \lfloor\tau_0 T\rfloor)\mathbb{I}(t > \lfloor\tau_0 T\rfloor)$, so that y_t incorporates a break in trend at time $\lfloor\tau_0 T\rfloor$ when $\beta_3 \neq 0$. Here, $\tau_0 \in [\tau_L, \tau_U] =: \Lambda \subset [0, 1]$; the quantities τ_L and τ_U are trimming parameters below and above which, respectively, a trend break is deemed not to occur. We assume that u_t is a zero mean, fractionally integrated process, with order of integration δ, denoted $I(\delta)$. Our interest centres on testing the null hypothesis $H_0 : \beta_3 = 0$ against the trend break alternative $H_1 : \beta_3 \neq 0$ in (2.1), without assuming knowledge of the value of δ.

The trend break model is completed by formalising the $I(\delta)$ properties of u_t as follows.

Assumption 1. For some process η_t which satisfies the conditions of Assumption 2 below, the process u_t is such that

$$u_t = \sum_{s=-\infty}^{t} \Delta_{t-s}^{(\delta)}\{\eta_s \mathbb{I}(s > 0)\}$$

with $\delta \in [0, 1/2) \cup (1/2, 3/2)$.

Here $\Delta_t^{(\delta)} := \Gamma(t+\delta)/(\Gamma(\delta)\Gamma(t+1))$, where $\Gamma(\cdot)$ denotes the Gamma function, such that $\Gamma(0) := \infty$ and $\Gamma(0)/\Gamma(0) := 1$. It is further assumed that $u_t = 0$ for $t \leq 0$.

Assumption 2. η_t is a causal and invertible finite order ARMA process with innovations ε_t that follow an independent, identically distributed [IID] process with $E(\varepsilon_t) = 0$, $E(\varepsilon_t)^2 = \sigma_\varepsilon^2$, and $E|\varepsilon_t|^q < \infty$, with $q > \max(4, \frac{2}{3-2\delta})$. Let $\sigma^2 := \sum_{h=-\infty}^{\infty} E(\eta_t \eta_{t+h})$ denote the long run variance (LRV) of η_t.

Remark 1. Both Assumptions 1 and 2, including the restriction that $\delta \neq 1/2$, are standard in the long memory literature. Assumption 1 is based on the definition of a Type II fractionally integrated process; see, for example, Marinucci and Robinson (1999). Assumption 2 places conditions on the distributed lags of η_t that are sufficient for Assumption A.2 of Marinucci and Robinson (2000) and for the FCLT for Type II fractionally integrated process described therein to hold.

Under the assumption that the order of integration parameter, δ, is either zero or one, HLT proposes a GLS-based trend break test, denoted t_λ therein, that uses a weighted average of two sup-Wald statistics; a level-based statistic appropriate for the case where $\delta = 0$ and a first difference-based statistic for the case where $\delta = 1$.[1] The weighting adjusts according to whether $\delta = 0$ or 1 and yields a statistic with the same critical values under H_0 in each case. Indeed the weight function is implicitly an estimator of δ when $\delta \in \{0, 1\}$. However, as shown in Iacone et al. (2011), under H_0, when $\delta \in (0, 1/2)$ or $\delta \in (1, 3/2)$, $t_\lambda \overset{p}{\rightarrow} \infty$, so that the HLT test will spuriously indicate the presence of a trend break with probability one in the limit, even though none exists. It is also shown that when $\delta \in (1/2, 1)$, $t_\lambda \overset{p}{\rightarrow} 0$, so its asymptotic size is zero.

Clearly then, t_λ does not represent a reliable test of the trend break hypothesis outside of $\delta \in \{0, 1\}$; however, the GLS-type motivation underpinning its construction can be extended to allow for more general δ and can be made operational using a suitable estimator of δ. To that end, consider the following δ-GLS transformed variant of (2.1):

$$\Delta^\delta y_t = \beta_1 \Delta^\delta\{1\mathbb{I}(t > 0)\} + \beta_2 \Delta^\delta\{t\mathbb{I}(t > 0)\} + \beta_3 \Delta^\delta DT_t(\tau_0) + \Delta^\delta u_t, \tag{2.2}$$

where Δ^δ is the fractional difference operator $\Delta^\delta := (1 - L)^\delta = \sum_{t=0}^{\infty} \Delta_t^{(-\delta)} L^t$ such that $\Delta^\delta u_t = \eta_t$. Notice that $\Delta^\delta y_1 = y_1$. Next define

$$x_t := [1, t, DT_t(\tau_0)]',$$
$$\Delta^\delta x_t := [\Delta^\delta\{1\mathbb{I}(t > 0)\}, \Delta^\delta\{t\mathbb{I}(t > 0)\}, \Delta^\delta DT_t(\tau_0)]'.$$

Observe that $\Delta^\delta x_1 = [1, 1, 0]'$ and that the third element of $\Delta^\delta x_{\lfloor\tau_0 T\rfloor+1}$ is equal to unity. Now define the following δ-GLS transformed regressand and regressors

$$y(\delta) := [\Delta^\delta y_1, \Delta^\delta y_2, \ldots, \Delta^\delta y_T]',$$
$$x(\delta, \tau_0) := [\Delta^\delta x_1, \Delta^\delta x_2, \ldots, \Delta^\delta x_T]'$$

and construct the OLS estimate of $[\beta_1, \beta_2, \beta_3]'$ in (2.2)

$$\hat{\beta}(\delta, \tau_0) := [x(\delta, \tau_0)' x(\delta, \tau_0)]^{-1} x(\delta, \tau_0)' y(\delta).$$

Our test of H_0 against H_1 is based on (a feasible version of) the Wald statistic for $\beta_3 = 0$; that is, for $R := [0, 0, 1]$,

$$\mathcal{W}(\delta, \tau_0, \sigma^2) := \sigma^{-2}\hat{\beta}(\delta, \tau_0)' R' [R[x(\delta, \tau_0)' x(\delta, \tau_0)]^{-1} R']^{-1} \times R\hat{\beta}(\delta, \tau_0).$$

Since the true putative break fraction, τ_0, is taken to be unknown, we follow the approach of Andrews (1993) and examine the associated sup-Wald type statistic

$$\mathcal{SW}(\delta, \sigma^2) := \sup_{\tau \in \Lambda} \mathcal{W}(\delta, \tau, \sigma^2).$$

[1] In fact HLT uses (positive) square root Wald statistics, but of course the inference will be identical.