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A decoupled approach for non-probabilistic reliability-based design optimization

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ABSTRACT

Non-probabilistic reliability-based design optimization (NRBDO) offers a powerful tool for structural design when uncertain-but-bounded parameters are considered. Like the reliability-based design optimization (RBDO), the NRBDO application for practical engineering structure is hindered by the huge computational effort involved in the repeated evaluation of non-probabilistic constraints. The decoupled strategy is one of the most efficient RBDO strategies. However, whether it is widely applicable for NRBDO problem remains unknown. In this paper, we make attempts to develop a decoupled strategy for NRBDO convex models based on the concerned performance approach, and propose a sequential optimization approach to handle the deterministic optimization and non-probabilistic reliability analysis sequentially. A new feasibility-checking criterion is further proposed, and the non-probabilistic constraints are divided into active, inactive and violated categories. Since only concerned points of active and violated constraints are calculated accurately, the computational cost associated with non-probabilistic constraint is decreased significantly. Four examples are tested to envelop representative NRBDO problems based on interval set, single-ellipse or multi-ellipse convex model. This way, we show that the decoupled strategies could be promising for a large variety of engineering NRBDO problems.

1. Introduction

There are many uncertainties stemming from practical engineering, such as geometric dimensions, material properties and external loads. All these inherent uncertain factors may lead to large variations of the structural properties and even failure. Thus, the non-deterministic structural optimization models, including reliability-based design optimization (RBDO) and robust design optimization (RDO), exhibit some attraction in both of theoretical research and practical applications [1–5]. Based on classical probability theory, RBDO and RDO approaches have been extensively studied in the methodology and applications, and the precise information on the probabilistic distribution of random variables should be provided for both RBDO and RDO [6,7]. Otherwise, the optimal design of RBDO or RDO may generate unacceptable error for practical applications [8,9]. Until now, the quantification of uncertain parameters still is a challenge for complex system, which seriously hinders the application of uncertain design optimization based on the probability theory [10].

Therefore, the non-probabilistic reliability-based design optimization (NRBDO) models, such as interval set model [11–13] and convex model [14–16], are suggested to deal with the non-deterministic structural analysis and optimization problems with respect to limited uncertain information. The purpose of convex model is to provide an effective tool for optimal design with uncertain-but-bounded parameters, which pays an extremely important role in non-probabilistic reliability analysis. In 1990s, Ben-Haim and Elishakoff [17–19] first introduced the concept of non-probabilistic reliability through convex model theory, which plays a well alternative role for RBDO when only a limit of information is available for uncertain factors. Then, Qiu and Elishakoff [20], Elishakoff et al. [21] suggested the interval set model for truss structures optimization with uncertain-but-bounded parameters. Majumder and Rao [22] developed an interval-based multi-objective optimization approach for design of aircraft wing structures. Although interval set model performs well for NRBDO, it suffers from the problem of interval extension. To this end, the parameterization method [23,24] was introduced. In general, NRBDO is effective for uncertain optimization when only limited experiment samples are applied for uncertain variables.

Similar to the RBDO, NRBDO convex model also is achieved by a nested, double loop optimization model, where the

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non-probabilistic constraints are calculated repeatedly at each iterative step of the deterministic optimization. Evidently, it will result in a daunting computational effort. Thus, how to reduce the number of function evaluations is crucial for NRBDO. Early attempts have been made by e.g. Lombardi and Haftka [25], who used the anti-optimization technique to alleviate the computational burden. Kang et al. [26] applied the concept of performance measure approach (PMA) of RBDO and introduced the concerned performance approach (CPA) to improve the efficiency of NRBDO approaches. Then, the linearization-based approach [27] and sequential approximate programming [28] are further constructed to deal with the hybrid models with probabilistic and non-probabilistic variables efficiently. Generally, the expensive computational cost has become an important problem as the development of NRBDO convex model, which hinders its applications seriously.

In order to improve the efficiency of NRBDO, the metamodels are also employed for NRBDO to substitute the actual objective and performance functions. Jiang et al. [29] applied the latin hypercube design (LHD) and response surface method for nonlinear interval model. Li et al. [30] combined the LHD and Kriging model to deal with the multi-objective optimal problem with uncertain-but-bounded problems. Khodaparast et al. [31] also suggested an iterative procedure based on the Kriging model for interval model. Recently, Yang et al. [32] applied the efficient global optimization for hybrid reliability model with probabilistic and non-probabilistic variables.

Decoupled approach has been deemed as one of the most promising strategy for solving RBDO problems in terms of accuracy, efficiency and robustness [1,33–35]. Until now, a series of non-probabilistic convex models have been developed, such as interval set, single-ellipse and multi-ellipse convex models, and the selection of these models should be determined by the experimental data [13]. However, there are few studies on developing the decoupled approach for these NRBDO convex models [27,28]. Thus, a universal decoupled strategy is urgently required to solve the associated NRBDO problems.

In this paper, a mathematic definition is given to distinguish the types of the NRBDO convex models (including interval set model, single-ellipse model and multi-ellipse model). Then, a new sequential optimization approach (SOA) is proposed based on CPA to reduce the number of function calls of NRBDO convex models, which performs the non-probabilistic reliability analysis and deterministic optimization sequentially. Moreover, a feasibility-checking criterion using the approximate or actual concerned performance value is constructed to identify the active or inactive constraint during the iterative process, and then the non-probabilistic constraints can be evaluated efficiently.

The outline of this paper is presented as follows: Section 2 introduces the basic concept of the non-probabilistic reliability-based design optimization. Then, the proposed method is described in detail in Section 3. Using a mathematical example, a ten truss structure, a welded beam and a stiffened shell design, the performance of the proposed method is demonstrated in Section 4. Finally, the concluding remarks are drawn in Section 5.

2. Non-probabilistic reliability-based design optimization

In this section, a review of NRBDO is presented. The formulation of NRBDO is formulated as

$$\begin{aligned}
&\text{find} \quad \mathbf{d}, \mathbf{x}^C \\
&\min_{\mathbf{d}} \quad C(\mathbf{d}, \mathbf{x}^C) \\
&\text{s.t.} \quad \eta_j(\mathbf{d}, \mathbf{x}, \mathbf{p}) \geqslant \underline{\eta}_j \quad j = 1, 2, \ldots, m \\
&\qquad \mathbf{d}^{\mathbf{L}} \leqslant \mathbf{d} \leqslant \mathbf{d}^{\mathbf{U}}
\end{aligned} \tag{1}$$

where C represents the objective function. $\mathbf{d}$ is the n_d-dimensional deterministic design variable vector, $\mathbf{x}$ and $\mathbf{p}$ are n_x-dimensional uncertain design variable vector and n_p-dimensional uncertain variable vector, respectively. $\mathbf{x}^C$ is the nominal value of $\mathbf{x}$. $\mathbf{x}^w$ is the radius of x, and is assumed unchanged during the iterative process. The η_j denotes the non-probabilistic reliability index with respect to target non-probabilistic reliability index $\underline{\eta}_j$, which can be obtained by

$$\begin{aligned}
&\text{find} \quad \mathbf{q} \\
&\max_{\mathbf{d}} \quad \eta = \mathrm{sgn}(g(\mathbf{0})) \cdot \min\left\{\max\left(\sqrt{\mathbf{q}_1^T\mathbf{q}_1}, \sqrt{\mathbf{q}_2^T\mathbf{q}_2}, \ldots, \sqrt{\mathbf{q}_k^T\mathbf{q}_k}\right)\right\} \\
&\text{s.t.} \quad g_i(\mathbf{q}) = 0 \quad i = 1, 2, \ldots, n_g
\end{aligned} \tag{2}$$

where $g_i(\cdot)$ is the ith performance function; n_g is the number of the performance functions; n_q is the number of grouped ellipsoid sets. All uncertain parameters ($\mathbf{x}$ and $\mathbf{p}$) should be transformed into the normalized uncertain parameters $\mathbf{q}$ in q-space, the details are shown in Ref. [26]. A example for a problem with 3 uncertain parameters is shown in Fig. 1. When $n_q = 3$, the multi-ellipsoid set model degenerates into a multi-dimensional interval set. When $n_q = 2$, the multi-ellipsoid set model is a hybrid convex model that is composed by an ellipsoid model and a set model. When $n_q = 1$, the multi-ellipsoid set model degenerates into a single-ellipsoid model.

2.1. Non-probabilistic reliability-index-based approach

The non-probabilistic reliability-index-based approach (NRIA) in Eq. (2) is a nested bi-level optimization structure, which is similar to the probabilistic RBDO problem. The outer loop of NRIA is a deterministic optimization, while the inner loop is an iterative process of non-probabilistic reliability index. Obviously, its calculation process is similar to that of the probabilistic reliability index approach (RIA), while the non-probabilistic reliability index can be solved by deterministic optimization approach.

2.2. Concerned performance approach

In order to improve the efficiency and robustness of NRBDO with reliability-index-based approach, the CPA is developed by adopting the concept of PMA [26]. The formulation of CPA for NRBDO is described as

$$\begin{aligned}
&\text{find} \quad \mathbf{d}, \mathbf{x}^C \\
&\min_{\mathbf{d}} \quad C(\mathbf{d}, \mathbf{x}^C) \\
&\text{s.t.} \quad \alpha_j(\mathbf{d}, \mathbf{x}, \mathbf{p}) \geqslant 0 \quad j = 1, 2, \ldots, m \\
&\qquad \mathbf{d}^{\mathbf{L}} \leqslant \mathbf{d} \leqslant \mathbf{d}^{\mathbf{U}}
\end{aligned} \tag{3}$$

where the $\alpha_j(\mathbf{d}, \mathbf{x})$ is the concerned performance of the jth constraint, and it is calculated by the following optimization model.

$$\begin{aligned}
&\text{find} \quad \mathbf{q} \\
&\min_{\mathbf{q}} \quad g_j(\mathbf{q}) \\
&\text{s.t.} \quad \mathbf{q}_i^T\mathbf{q}_i \leqslant \underline{\eta}_j^2 \quad i = 1, 2, \ldots, nq
\end{aligned} \tag{4}$$

where the optimum $\mathbf{q}_{\mathrm{CP}}$ is defined as the concerned point (CP), and $g_j(\cdot)$ is the jth performance function. The optimization model of Eq. (4) searches for the minimum value of performance function at the multiple hyper-sphere in q-space. Since the CPA has more simple constraint than NRIA, it performs well in terms of efficiency and robustness.