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Modelling volatility by variance decomposition

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ABSTRACT

In this paper, we propose two parametric alternatives to the standard GJR-GARCH model of Glosten et al. (1993), based on additive and multiplicative decompositions of the variance. They allow the variance of the model to have a smooth time-varying structure. The suggested parameterizations describe structural change in the conditional and unconditional variances where the transition between regimes over time is smooth. The main focus is on the multiplicative decomposition of the variance into an unconditional and conditional components. Estimation of the multiplicative model is discussed in detail. An empirical application to daily stock returns illustrates the functioning of the model. The results show that the 'long memory type behaviour' of the sample autocorrelation functions of the absolute returns can also be explained by deterministic changes in the unconditional variance.

1. Introduction

Modelling time-varying volatility of financial returns has been a flourishing field of research for a quarter of a century following the introduction of the Autoregressive Conditional Heteroskedasticity (ARCH) model by Engle (1982) and the Generalized ARCH (GARCH) model developed by Bollerslev (1986). These basic models have since been generalized in many ways; see Teräsvirta (2009) for a recent survey. The increasing popularity of GARCH models has been mainly due to their ability to describe the dynamic structure of volatility clustering of stock return series, specifically over short periods of time. However, one may expect that economic or political events or changes in institutions cause the structure of volatility to change over time. This means that the assumption of stationarity may be inappropriate under the evidence of structural changes in financial return series. Mikosch and Stărică (2004) argued that stylized facts in financial return series such as the long-range dependence and the 'integrated GARCH effect' can be well explained by unaccounted structural breaks in the unconditional variance; see also Lamoureux and Lastrapes (1990). Diebold (1986) was the first to suggest that occasional level shifts in the intercept of the GARCH model can bias the estimates towards the parameters of an integrated GARCH model.

Another line of research has focused on explaining nonstationary behaviour of volatility by long-memory models, such as the Fractionally Integrated GARCH (FIGARCH) model by Baillie et al. (1996). The FIGARCH model is not the only way of handling the 'integrated GARCH effect' in return series. Baillie and Morana (2009) generalized the FIGARCH model by allowing a deterministically changing intercept. Hamilton and Susmel (1994) and Cai (1994) suggested a Markov-switching ARCH model for the purpose, and their model has later been generalized by others. One may also assume that the GARCH process contains sudden deterministic switches and try and detect them; see Berkes et al. (2004) who proposed a method of sequential switch or change-point detection.

Yet another way of dealing with high persistence would be to explicitly assume that the volatility process is 'smoothly' nonstationary and model it accordingly. Dahlhaus and Subba Rao (2006) introduced a time-varying ARCH process for modelling nonstationary volatility. Their tvARCH model is asymptotically locally stationary at every point of observation but it is globally nonstationary because of time-varying parameters. van Bellegem and von Sachs (2004) and, later, Engle and Rangel (2008) assumed that the variance of the process of interest can be decomposed into

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two components, a stationary and a nonstationary one. The former authors fitted the deterministic component nonparametrically to the squared observations, whereas the latter described the nonstationary component by using splines. Both assumed that the stationary component follows a GARCH process. For a similar approach using a different version of splines, see Brownlees and Gallo (2010). Mishra et al. (2010) did not explicitly mention nonstationarity but used the multiplicative decomposition to correct the potential misspecification due to a 'rough' parametric GARCH specification by a smooth nonparametric component. Yet another multiplicative decomposition was introduced in Osiewalski (2009) and Osiewalski and Pajor (2009). Some of these developments are described in detail in van Bellegem (2012) and Teräsvirta (2012).

In this paper, we introduce two nonstationary GARCH models for situations in which volatility appears to be nonstationary. First, we propose an additive time-varying parameter model, in which a directly time-dependent component is added to the GJR-GARCH specification. In the second alternative, the variance is multiplicatively decomposed into the stationary and nonstationary component as in van Bellegem and von Sachs (2004) or Engle and Rangel (2008). The deterministic component of the variance is parametric and thus different from previous approaches. We show that the multiplicative decomposition is a special case of the general additive decomposition. As we shall see, this component is very flexible and can describe many types of nonstationary behaviour. The specification and evaluation issues involved are discussed separately in Amado and Teräsvirta (2012). In this work we concentrate on parameter estimation.

The outline of this paper is as follows. In Section 2 we present the new Time-Varying (TV-) GARCH or GJR-GARCH model and highlight some of its properties. Maximum likelihood estimation of the model is discussed in Section 3. Section 4 contains an empirical example to a daily stock index return series. Finally, Section 5 contains concluding remarks.

2. The model

Let the model for an asset or index return y_t be

$$y_t = \mu_t + \varepsilon_t$$

where $\{\varepsilon_t\}$ is an innovation sequence with the conditional mean $\mathsf{E}(\varepsilon_t|\mathcal{F}_{t-1}) = 0$ and a potentially time-varying conditional variance $\mathsf{E}(\varepsilon_t^2|\mathcal{F}_{t-1}) = \sigma_t^2$, and $\mathcal{F}_{t-1}$ is the σ-field generated by the available information until $t-1$. We assume that $\mathsf{E}(y_t|\mathcal{F}_{t-1}) = 0$ and focus solely on σ_t^2. Let

$$\varepsilon_t = \zeta_t \sigma_t \tag{1}$$

where $\{\zeta_t\}$ is a sequence of independent random variables with mean zero and variance one. Furthermore, assume that σ_t^2 is a time-varying representation measurable with respect to $\mathcal{F}_{t-1}$ with either an additive structure

$$\sigma_t^2 = h_t + g_t \tag{2}$$

or a multiplicative one

$$\sigma_t^2 = h_t g_t. \tag{3}$$

The function h_t is a component describing conditional heteroskedasticity in the observed process y_t, whereas g_t introduces nonstationarity. Since we are going apply our model to stock return series, where asymmetry of the response to shocks becomes an issue, we assume that h_t follows the stationary GJR-GARCH(p, q) model of Glosten et al. (1993):

$$h_t = \alpha_0 + \sum_{i=1}^{q}(\alpha_i + \lambda_i I(\varepsilon_{t-i} < 0))\varepsilon_{t-i}^2 + \sum_{j=1}^{p}\beta_j h_{t-j} \tag{4}$$

where $I(A)$ is the indicator variable: $I(A) = 1$ when A is true, and zero otherwise. Alternatively, we could use the asymmetric GARCH model of Engle and Ng (1993) and the quadratic GARCH model by Sentana (1995). The GJR-GARCH(p, q) model is nested in (2) when $g_t \equiv 0$ and in (3) when $g_t \equiv 1$. Note that when (3) holds, ε_{t-i}^2 is replaced by $\phi_{t-i}^2 = \varepsilon_{t-i}^2/g_{t-i},\ i = 1, \ldots, q$, in (4). Both (2) and (3) combined with (1) define a time-varying parameter GARCH model.

In order to characterize smooth changes in the conditional variance we assume that the parameters in (4) vary smoothly over time. This is done for example by defining the function g_t in (2) as follows:

$$g_t = \left\{\alpha_0^* + \sum_{i=1}^{q}(\alpha_i^* + \lambda_i^* I(\varepsilon_{t-i} < 0))\varepsilon_{t-i}^2 + \sum_{j=1}^{p}\beta_j^* h_{t-j}\right\} \times G(t/T; \gamma, \mathbf{c}), \tag{5}$$

where $G(t/T; \gamma, \mathbf{c})$ is the so-called transition function which is a continuous and non-negative function bounded between zero and one. Furthermore, time t/T is the transition variable and is defined on the interval [0, 1], where T is the number of observations. A suitable choice for $G(t/T; \gamma, \mathbf{c})$ is the general logistic transition function

$$G(t/T; \gamma, \mathbf{c}) = \left(1 + \exp\left\{-\gamma \prod_{k=1}^{K}(t/T - c_k)\right\}\right)^{-1}, \quad \gamma > 0,\ c_1 \leq \cdots \leq c_K. \tag{6}$$

The idea of applying smooth transition to modelling parameter change with (6) was considered by Lin and Teräsvirta (1994). This function is such that the parameters of the GJR-GARCH model (1)–(2) fluctuate smoothly over time between $(\alpha_i, \lambda_i, \beta_j)$ and $(\alpha_i + \alpha_i^*, \lambda_i + \lambda_i^*, \beta_j + \beta_j^*),\ i = 0, 1, \ldots, q,\ j = 1, \ldots, p$. The slope parameter γ controls the degree of smoothness of the transition function. When $\gamma \longrightarrow \infty$, the switch from one set of parameters to another in (2) is abrupt, that is, the process contains structural breaks at $c_1, c_2, \ldots, c_K$. The order $K \in \mathbb{Z}_+$ determines the shape of the transition function. Typical choices for the transition function in practice are $K = 1$ and $K = 2$. These are illustrated in Fig. 1 for a set of values for γ, c_1, and c_2. Large values of γ increase the velocity of transition from 0 to 1 as a function of t/T. The TV-GARCH model with $K = 1$ is suitable for describing return processes whose volatility dynamics are different before and after the smooth structural change. When $K = 2$, the parameters change but eventually return towards their original values as a function of time.

More generally, one can define an extended version of the additive TV-GJR-GARCH model by allowing more than one transition function in g_t. The result becomes

$$g_t = \sum_{l=1}^{r}\left\{\alpha_{0l} + \sum_{i=1}^{q}(\alpha_{il} + \lambda_{il} I(\varepsilon_{t-i} < 0))\varepsilon_{t-i}^2 + \sum_{j=1}^{p}\beta_{jl} h_{t-j}\right\} G_l(t/T; \gamma_l, \mathbf{c}_l) \tag{7}$$

where $G_l(t/T; \gamma_l, \mathbf{c}_l),\ l = 1, \ldots, r$, are logistic functions as in (6) with smoothness parameter γ_l and a threshold parameter vector $\mathbf{c}_l$. The parameters in (4) and (7) satisfy the restrictions $\alpha_0 + \sum_{l=1}^{j}\alpha_{0l} > 0, \alpha_i + \lambda_i/2 + \sum_{l=1}^{j}(\alpha_{il} + \lambda_{il}/2) > 0,\ i = 1, \ldots, q$, and $\beta_i + \sum_{l=1}^{j}\beta_{il} \geq 0,\ i = 1, \ldots, p$, all for any $j \in \{1, \ldots, r\}$. These conditions are sufficient for $g_t > 0$ for all t.

The model (1), (2), (4) and (5) or, more generally (7), is an additive TV-GJR-GARCH model whose intercept, ARCH and GARCH parameters are all time-varying. This implies that the model is capable of accommodating systematic changes both in the 'baseline volatility' (or unconditional variance) and in the