Journal of Econometrics 174 (2013) 1–14

Contents lists available at SciVerse ScienceDirect

Journal of Econometrics

journal homepage: www.elsevier.com/locate/jeconom

Panel data models with multiple time-varying individual effects

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ARTICLE INFO

Article history:
Received 20 October 2006
Received in revised form
3 December 2012
Accepted 19 December 2012
Available online 8 January 2013

JEL classification:
C51
D24

Keywords:
Panel data
Time-varying individual effects
Factor models

ABSTRACT

This paper considers a panel data model with time-varying individual effects. The data are assumed to contain a large number of cross-sectional units repeatedly observed over a fixed number of time periods. The model has a feature of the fixed-effects model in that the effects are assumed to be correlated with the regressors. The unobservable individual effects are assumed to have a factor structure. For consistent estimation of the model, it is important to estimate the true number of individual effects. We propose a generalized methods of moments procedure by which both the number of individual effects and the regression coefficients can be consistently estimated. Some important identification issues are also discussed. Our simulation results indicate that the proposed methods produce reliable estimates.

1. Introduction

The use of panel data has been increasingly popular in empirical microeconomic and macroeconomic studies. An important advantage of panel data is that researchers can obtain consistent estimates of important parameters while controlling for unobservable cross-sectional heterogeneity. An example of such heterogeneity, the so-called *individual effect*, is the effect of talent in a model of workers' hourly earnings. In order to estimate the effect of education on the hourly wage rate consistently, researchers need to control for heterogeneity in workers' talents or skills. Unfortunately, data containing information on individual workers' talents and skills are extremely rare. Without such information, it is extremely difficult, if not impossible, to control for talent using cross-sectional data. In contrast, when panel data are available, a variety of estimation methods (e.g., Hausman and Taylor, 1981; Amemiya and MaCurdy, 1986; Cornwell and Rupert, 1988; Breusch et al., 1989) can be used to control for the unobservable individual effects. Even if individual workers' talents are unobservable, it is possible to estimate the effect of education on the hourly wage consistently.

Standard panel data models such as those cited above assume that the unobservable individual effect is a single time-invariant component. However, this assumption may be excessively restrictive. For example, consider a model of hourly wage rates. It is well-known that labor productivity changes over the business cycle. Accordingly, the productivity of an individual's unobservable talent or skill would also change over the business cycle (Ahn et al., 2001). If so, the effect of unobservable talent on hourly wages would vary over time because workers' hourly wage rates depend on their labor productivity. It is also likely that hourly wage rates depend on multiple individual effects. For example, individual workers' wages could be affected by unexpected changes in macroeconomic variables. However, the effects of these aggregate variables on wages would depend on individual-specific characteristics such as the worker's residential area and occupation. Panel data models that assume a single time-invariant individual effect are inappropriate for the analysis of data with such multiple time-varying individual effects.

There are many other examples of models that may require multiple time-varying effects. One example is the consumption model based on the life-cycle and rational-expectation hypothesis. This model predicts that current consumption growth depends on the unobservable marginal utility of expected life-time wealth. When consumers' future incomes are uncertain, their marginal utility of wealth varies over time (Altug and Miller, 1990; Pischke, 1995). Another example is asset pricing models that assume time-varying risk premia (Campbell, 1987; Ferson and Foerster,

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doi:10.1016/j.jeconom.2012.12.002

1994; Zhou, 1994). These models can be also viewed as panel data models with unobservable multiple time-varying individual effects. Finally, our approach can be used for the empirical studies of economic growth based on international data (e.g., Mankiw et al., 1992; Islam, 1995; Caselli et al., 1996). Individual countries' economic growth rates could depend on world-wide supply shocks (such as the oil shocks in the 1970's) and also on technology shocks (such as the rapid development of the information technology industry in the 1990's). However, the effect of such world-wide shocks could depend on country-specific factors such as available human capital and natural resources.

In this paper we consider the model

$$y_{it} = x_{it}'\beta + \Sigma_{j=1}^{p}\xi_{tj}\alpha_{ij} + \varepsilon_{it}. \tag{1}$$

Here $i = 1, \ldots, N$, $t = 1, \ldots, T$, and we consider only the case that N is large and T is small (so asymptotic analyses are as $N \to \infty$ with T fixed). The x_{it} are $k \times 1$ vectors of regressors. The ξ_{tj} are unobservables that affect everyone, but because the α_{ij} vary over i, they do not affect everyone equally. The ε_{it} are idiosyncratic noise (random errors).

For reasons given below, we will try to avoid the terminology of factor models. We will refer to the ξ_{tj} as "macro shocks" (as opposed to "factors") and to the α_{ij} as "random coefficients" (as opposed to "factor loadings"). The product $\xi_{tj}\alpha_{ij}$ is the ith person's response to the jth macro shock, and we will refer to it as "time-varying individual effect". Our model contains p time-varying individual effects.

There is a huge literature on panel data models with error components indexed by both i and t. Perhaps the earliest treatment is the two-way error components structure in which $u_{it} = \alpha_i + \xi_t + \varepsilon_{it}$, which is a special case of our Eq. (1). This two-way model comes in a random effects version, in which the components α_i and ξ_t are random and uncorrelated with the regressors, and a fixed effects version, in which the components are treated as fixed parameters, and so their correlation with the regressors is unrestricted. The point of a random effects model is to account for correlations over time or across individuals, while the point of a fixed effects model is to control for unobservables. The two-way model was introduced in a series of articles including Balestra and Nerlove (1966), Wallace and Hussain (1969), Amemiya (1971) and Fuller and Battese (1974). Kmenta (1986, Section 12.2) proposed some alternative random effects models that allowed for a rich pattern of correlations over time or across individuals. For example, he has a model with cross sectional heteroskedasticity and time series autocorrelation (of AR(1) form), and another model with cross sectional correlation (but homoskedasticity) and time series autocorrelation. He also discusses the random effects and fixed effects two-way models. These models were further analyzed by Baltagi (1986) and Baltagi et al. (1992). See also Baltagi (2001, Chapters 3 and 5). Pesaran and Smith (1995) suggest a random coefficient model that leads to interactions between individual-specific effects and time-varying variables, but in this case (and unlike in our Eq. (1)) the time-varying variables are observable.

In our model, we will treat the ξ_{tj} as fixed parameters and the α_{ij} as random. However, the motivation for our model will be similar to that of a fixed effects model, namely, to avoid bias by controlling for unobservables. Specifically, an important feature of our model is that the α_{ij} are allowed to be correlated with some or all of the variables in x_{it}. Thus we will need to control for the unobservable effects in order to estimate β consistently. One possibility would be a fixed effects model in which both the ξ_{tj} and α_{ij} are estimated as parameters. However, from Ahn et al. (2001, hereafter ALS) and Bai (2003), it is known that, in the "large N, small T" case, the consistency of this estimator depends on the ε_{it} being white noise, which we do not wish to assume. As a result, we obtain identification differently, by assuming that the regressors x_{it} may be correlated with the α's but not correlated with the ε's.

Panel data models with time-varying individual effects and small numbers of time-series observations have previously been studied by Holtz-Eakin et al. (1988), Lee (1991), Chamberlain (1992), and ALS. However, these studies, except Lee (1991), consider only the case of a single individual effect. Lee (1991) considered the case of multiple individual effects, but he made the assumption that the errors ε_{it} are white noise, and he assumed that the true number of individual effects was known.

The composite error $u_{it} = \Sigma_{j=1}^{p}\xi_{tj}\alpha_{ij} + \varepsilon_{it}$ has an "exact factor structure" in the sense that algebraically it has the same structure as is assumed in the classical treatment of factor analysis; e.g., Anderson (1984, Chapter 14). Classical factor analysis is designed to account for the correlations between different elements of the observable variable (which corresponds algebraically to our composite error). According to Anderson, p. 553, the "crucial assumption (of an exact factor model) is that the components of U (our ε) are uncorrelated", so that correlations across variables are due to the common factors. The composite error structure we consider in this paper is algebraically the same as an exact factor structure, but the errors ε_{it} are allowed to be heteroskedastic and/or autocorrelated over time, so that the $T \times T$ variance matrix of the ε_{it} and therefore of the u_{it} is completely unrestricted. Exact factor models are restrictive compared to the "approximate factor models" that allow cross-sectional correlation in the errors ε_{it}; see, for example, Chamberlain and Rothschild (1983), Connor and Korajczyk (1993), Bai and Ng (2002) and Bai (2003). The estimators we propose in this paper would be consistent even if the errors ε_{it} were cross-sectionally correlated, although we do not consider such cases. Consistent estimation of the estimators' asymptotic variance matrices would require information on the cross-sectional correlation structure. A detailed discussion of possible cross-sectional correlation structures deserves a separate study.

The model we consider is different from both the exact and approximate factor models in that it contains observable regressors x_{it}. The studies mentioned above focus on how to estimate the number of common macro shocks and/or (linear combinations of) the shocks themselves. While estimation of these shocks is also important in our study, our major concern in this paper is how to consistently (and possibly efficiently) estimate the coefficient vector β. The model is related to a single-period cross-sectional regression model considered by Andrews (2005), in which data are cross-sectionally correlated through some common shocks like our ξ_{tj}. For the one-period model, Andrews shows that the least squares estimator is inconsistent unless regressors and disturbances are uncorrelated conditionally on the unobservable common shocks. Eq. (1) is an extension of his model to a multiple period model. Unless the random coefficients α_{ij} and the regressors x_{it} are uncorrelated conditionally on the shocks ξ_{tj}, the least squares or within estimators of β are inconsistent.

Bai (2009) and Kneip et al. (2012) also have considered consistent estimation of Eq. (1), but for the cases in which both N and T are large. They also propose consistent estimators of β in the presence of heterogeneity that is correlated with the regressors. Bai (2009) uses a nonlinear least squares estimator that would not be consistent when N is large but T is small. Kneip et al. (2012) considered a model similar to that of Bai (2009), but with the additional assumption that the factors change slowly and smoothly over time. In this paper we achieve identification via an additional assumption that these papers did not have to make, that there are observable variables correlated with the random coefficients α_{ij} but not with the idiosyncratic errors ε_{it}.

In summary, compared to the existing literature, this paper has the following distinctive features. First, it focuses on the correlation between the random coefficients and the regressors. Second, it