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Semi-parametric estimation of American option prices

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ABSTRACT

We introduce a novel semi-parametric estimator of American option prices in discrete time. The specification is based on a parameterized stochastic discount factor and is nonparametric w.r.t. the historical dynamics of the Markovian state variables. The historical transition density estimator minimizes a distance built on the Kullback–Leibler divergence from a kernel transition density, subject to the no-arbitrage restrictions for a non-defaultable bond, the underlying asset and some American option prices. We use dynamic programming to make explicit the nonlinear restrictions on the Euclidean and functional parameters coming from option data. We study asymptotic and finite sample properties of the estimators.

1. Introduction

This paper deals with the estimation of American option prices in a discrete time, incomplete market, Markovian framework. The state variables vector includes the return on the fundamental asset and other relevant pricing factors, such as the asset stochastic volatility and the discount rate. An American option differs from the corresponding European security since the holder has the right to exercise the option on or before the maturity date (see Broadie and Detemple (2004) and Detemple (2005), for reviews on valuation of American-style derivatives). Thus, the American option valuation problem can be faced as an optimal stopping time problem (see Bensoussan (1984) and Karatzas (1988, 1989)).[1] Equivalently, at each date the option value is the maximum between the exercise payoff and the continuation value, that is, the risk adjusted and time discounted conditional expectation of the one-period-ahead option value. This dynamic programming argument suggests that, in a discrete time framework, the pricing of an American option can be represented by a backward recursive application of a valuation operator that embodies both the exercise decision and the computation of the continuation value.

The literature on dynamic programming approaches to American option pricing has mostly focused on parametric models for the risk-neutral dynamics of the state variables vector, such as the Black–Scholes, stochastic volatility and jump-diffusion models. The time is discretized and, for given values of the model parameters, the backward recursive option valuation is performed assuming a finite set of possible values for the state variables at each date. In lattice methods the state variables domain is discretized in a deterministic way depending on the model (see, e.g., the binomial tree of Cox et al. (1979), the trinomial tree of Boyle (1988), the multinomial tree of Kamrad and Ritchken (1991), and the efficient lattice algorithm in Ritchken and Trevor (1999)). In Monte Carlo methods the state variables domain is discretized in a stochastic way based on a special choice of the space sampling (see, e.g., the random tree of Broadie and Glasserman (1997), the regression-based Monte Carlo methods of Carriere (1996), Longstaff and Schwartz (2001) and Tsitsiklis and Van Roy (2001), and the stochastic mesh of Broadie and Glasserman (2004)). For instance, in regression-based Monte Carlo methods a sample of state variables paths is artificially generated from the model. The conditional expectation that gives the continuation value at a given

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[1] Alternative characterizations of the American option pricing problem for special parameterizations of the state variables process include for instance the free boundary formulation (see, e.g., McKean (1965), Brennan and Schwartz (1977), Barone-Adesi and Whaley (1987) and Huang et al. (1996)).

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date and state is approximated by using nonparametric regression methods applied to the simulated cash-flows or option values at the future dates. Glasserman (2004) explains how regression-based Monte Carlo methods can be interpreted as stochastic mesh approaches.

Despite this huge body of literature on valuation, the analysis of statistical estimation methods with American option price data is very limited, likely because of the complexity induced by the pricing problem. Nonparametric estimation methods are particularly convenient in this respect, since they allow to bypass this complexity by postulating a flexible link function relating the American option price with observable contract characteristics and state variables. For instance, Broadie et al. (2000a,b) consider kernel-based regression methods including the moneyness strike, the time-to-maturity, the asset stochastic volatility and dividend yield among the regressors. In an empirical study, these authors find that both dividend yield and stochastic volatility are important determinants of the American option price. Other nonparametric approaches, such as splines and neural networks, are also possible (see Daglish (2003), for a comparative study as well as Hutchinson et al. (1994) and Garcia and Gencay (2000), for the use of neural networks to price European options).

We depart from this literature by combining the dynamic programming formulation with a semi-parametric specification of the risk-neutral distribution in discrete time. Specifically, the historical transition density f of the Markov state is left unconstrained and treated as a functional parameter, while the Stochastic Discount Factor (SDF) is assumed to be in a parametric family indexed by the finite-dimensional parameter θ. The goal is to estimate the true values f_0 and θ_0 of the model parameters by the information in a time-series of state variables observations and a cross-section of observed American option prices. The estimates of θ_0 and f_0 are used to estimate the prices of American options that are not actively traded on the market at the current time. We also propose new semi-parametric estimators for a class of linear or nonlinear functionals of θ and f that include historical and risk-neutral conditional cross-moments of the state variables, such as leverage effects (see Black (1976)) and term structures of skewness and kurtosis measures (e.g., Bakshi et al. (2003)).

The semi-parametric setting introduced in this paper is intermediate between fully parametric and fully nonparametric approaches. The advantage w.r.t. the former approach is the flexibility in modeling the historical transition density, which allows to get estimators of the option prices and exercise boundary in a rather general model setting. Moreover, we get a proper distribution theory for the estimators without introducing ad-hoc pricing errors. The advantage w.r.t. the latter approach is that the estimated pricing model is arbitrage-free. In nonparametric approaches, ensuring the absence of arbitrage opportunities by imposing shape restrictions on the pricing function might be difficult, since such shape restrictions are not completely known for American options in a general framework (see, e.g., Aït-Sahalia and Duarte (2003), Yatchew and Härdle (2006) and Birke and Pilz (2009), for constrained nonparametric estimation of the state price density from European option data).

The information contained in the historical state variables and cross-sectional option data is exploited through the associated no-arbitrage restrictions. In our framework these restrictions are multi-period and involve the recursive valuation operator for American options. The resulting constraints on θ_0 and f_0 are nonlinear w.r.t. both parameters and do not correspond to standard moment restrictions. This feature yields a setting that is different from the ones of the Generalized Method of Moments (GMM, see Hansen (1982) and Hansen and Singleton (1982)), the Extended Method of Moments (XMM, see Gagliardini et al. (2011)) and other semi-parametric settings considered in the literature (e.g., Ai and Chen (2003); see also Powell (1994) and Ichimura and Todd (2007), for reviews). This difference explains the methodological novelty of our paper. To get numerically tractable estimators, we consider a two-step approach. First, the SDF parameter θ_0 is estimated by minimizing a distance criterion that corresponds to a quadratic form of the empirical constraint vector. Second, the historical transition density f_0 is estimated by minimizing an information-theoretic criterion subject to the set of no-arbitrage restrictions with estimated SDF parameter. The information criterion is based on the Kullback–Leibler distance of f_0 from a kernel density estimator (see Kitamura and Stutzer (1997) and Kitamura et al. (2004)).

Despite the differences in terms of model specification and data usage, comparing our estimation methodology with the existing literature on dynamic programming valuation gives interesting insights. Indeed, for any given value of the SDF parameter vector θ, we compute the conditional expectation that gives the continuation value as a weighted average over the sample observations of the state variables. Thus, our approach is closer in spirit to stochastic mesh than to lattice methods, with the historical realization of the state variables vector process taken as a mesh. The weights turn out to be kernel weights adjusted by a tilting factor accounting for the no-arbitrage restrictions, and multiplied by the SDF to pass from the historical to the risk-neutral distribution.

In Section 2 we describe the discrete time Markovian framework and define the American option pricing operator for recursive valuation. In Section 3 we introduce the semi-parametric specification with historical transition density f of the state variables and SDF parameter θ. We discuss the no-arbitrage restrictions from the available historical and option data. We investigate the local sensitivity of the no-arbitrage constraint vector to the model parameters by computing the gradient of the constraints w.r.t. θ and their Fréchet derivative w.r.t. f. In Section 4 we introduce the semi-parametric estimators of the true SDF parameter θ_0, the true historical transition density f_0 and a class of their functionals, including the American option prices. We study the large sample properties of these estimators in Section 5. The asymptotics is for a long time-series of state variables observations and a fixed number of cross-sectionally observed option prices. We link the asymptotic properties of the proposed estimators to the ones of information-theoretic GMM estimators, by interpreting the Fréchet derivative of the constraint vector as a moment function locally around the true transition density f_0. In Section 6 we present the results of a Monte Carlo experiment to study the finite-sample properties of the estimators. Section 7 concludes. In Appendix A we list the set of regularity assumptions for the validity of the asymptotic properties. Proofs of the propositions are gathered in Appendices B–F and proofs of technical lemmas in supplementary materials available on our web-pages.

2. Valuation of American options

In this section we define the dynamics of the state variables and asset prices. We first consider the state variables and the SDF in Section 2.1. We then state an homogeneity property w.r.t. the underlying asset price for a class of American options in Section 2.2. Finally in Section 2.3 we introduce an operator formulation for the American option price useful for the derivation of the theoretical results.

2.1. The framework

We consider an incomplete market framework in discrete time. The time index t, with $t \in \mathbb{N}$, identifies a trading day. A fundamental asset (a stock, say) with price S_t, a short-term non-defaultable zero-coupon bond and a set of American options