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Chi-squared tests for evaluation and comparison of asset pricing models

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ABSTRACT

This paper presents a general statistical framework for estimation, testing and comparison of asset pricing models using the unconstrained distance measure of Hansen and Jagannathan (1997). The limiting results cover both linear and nonlinear models that could be correctly specified or misspecified. We propose modified versions of the existing model selection tests and new pivotal specification and model comparison tests with improved finite-sample properties. In addition, we provide formal tests of multiple model comparison. The excellent size and power properties of the proposed tests are demonstrated using simulated data from linear and nonlinear asset pricing models.

1. Introduction

It is common for financial economists to view all asset pricing models only as approximations to reality. Although these models are likely to be misspecified, it is still useful to empirically evaluate the degree of misspecification and their relative pricing performance using actual data. In their seminal paper, Hansen and Jagannathan (1997, HJ hereafter) propose measures of model misspecification that are now routinely used for parameter estimation, specification testing and comparison of competing asset pricing models. The unconstrained (constrained) HJ-distance measures the distance between the stochastic discount factor (SDF) of a proposed model and the set of (nonnegative) admissible stochastic discount factors. But despite the recent advances in developing the appropriate econometric theory for comparing asset pricing models based on the HJ-distance, a general statistical procedure for model selection in this context is still incomplete. As a result, many researchers are still ranking alternative models by comparing their corresponding sample HJ-distances without any use of a formal statistical criterion that takes into account the sampling and model misspecification uncertainty. In this paper, we provide a comprehensive statistical framework for estimation, evaluation and comparison of linear and nonlinear (potentially misspecified) asset pricing models based on the unconstrained HJ-distance. Given some unappealing theoretical properties of the constrained HJ-distance (Gospodinov et al., 2011), we do not consider explicitly the sample constrained HJ-distance but the generality of our analytical framework allows us to easily extend the main results for the unconstrained HJ-distance that we derive in this paper to its constrained analog (a detailed econometric analysis of the sample constrained HJ-distance is available from the authors upon request). Our framework could also be used to study the statistical properties of other measures of model misspecification.

The econometric methodology for using the unconstrained HJ-distance as a specification test for linear and nonlinear models is developed by Hansen et al. (1995), Jagannathan and Wang (1996) and Parker and Julliard (2005). Kan and Robotti (2009) provide a statistical procedure for comparing linear asset pricing models based on the unconstrained HJ-distance. Furthermore, Kan and Robotti (2009) propose standard errors for the SDF parameter

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estimates that are valid for misspecified models. Almeida and Garcia (2012) consider estimation and inference in SDF models based on more general minimum discrepancy measures of model misspecification. The objective of this paper is to provide a unifying framework for improved statistical inference, specification testing and (pairwise and multiple) model comparison based on the sample HJ-distances of competing linear and nonlinear asset pricing models.

Our main contributions can be summarized as follows. First, we propose a new Lagrange multiplier test for correct model specification. This new specification test is asymptotically chi-squared distributed and enjoys improved finite-sample properties compared to the specification test based on the HJ-distance. Second, we derive the non-degenerate joint asymptotic distribution of the parameters and the Lagrange multipliers which are not always asymptotically normally distributed.[1] Third, we improve upon the model selection testing procedures in the existing literature. This is achieved by incorporating the appropriate null hypotheses which leads to simpler model comparison tests that require the estimation of far fewer parameters than the existing testing procedures. While the practice of not imposing the null hypotheses in constructing the test statistics can be justified based on asymptotic arguments, it produces the undesirable outcome of comparing test statistics that are positive by construction (as in the nested model case discussed in Section 3) to distributions that can take on negative values. Our modifications are new to the literature on model selection tests and lead to substantial power improvements in situations with many test assets (moment conditions). Importantly, the proposed tests can be easily adapted to other setups including the quasi-likelihood framework of Vuong (1989). Fourth, we propose pivotal (asymptotically chi-squared distributed) versions of the model comparison tests that are easier to implement and analyze than their weighted chi-squared counterparts. The chi-squared tests appear to possess excellent finite-sample properties and their improved power proves to be particularly important in situations where they are used as pre-tests in sequential testing procedures for non-nested models. Fifth, we develop tests for multiple model comparison as well as fast numerical algorithms for computing their asymptotic p-values.[2] Finally, we investigate the finite-sample performance of the proposed inference procedures using simulated data from some popular linear and nonlinear asset pricing models.

The rest of the paper is organized as follows. Section 2 introduces the population and sample HJ-distance problems. It also presents the basic assumptions and the asymptotic properties of the sample HJ-distance and its corresponding estimators. Section 3 develops our pairwise and multiple model comparison tests based on the sample HJ-distances. Section 4 studies the finite-sample properties of our testing procedures using Monte Carlo simulation experiments. Section 5 concludes. The proofs of the main results in the paper are collected in the Appendix B. Some additional theoretical and simulation results are provided in an online appendix available on the authors' websites.

The paper adopts the following notation. Let $\overset{A}{\sim}$ stand for "asymptotically distributed as", χ^2_p signify a chi-squared random variable with p degrees of freedom, $|w| = (w'w)^{\frac{1}{2}}$ denote the Euclidean norm of a vector w and $\|A\| = \sqrt{\text{tr}(A'A)}$ be the Euclidean or Frobenius norm of a matrix A, where $\text{tr}(\cdot)$ is the trace operator. Finally, let $Z = (Z_1, \ldots, Z_s)'$ be a vector of s independent standard normal random variables, and let $\xi = (\xi_1, \ldots, \xi_s)'$ be a vector of s real numbers. Then, $F_s(\xi) = \sum_{i=1}^{s} \xi_i Z_i^2$ denotes a random variable which is distributed as a weighted sum of s independent chi-squared random variables with one degree of freedom.

[1] A more complete analysis of this problem is presented in Gospodinov et al. (2012).

[2] The Matlab codes for implementing all the statistical tests and procedures discussed in the paper are available upon request.

2. Estimation and model evaluation based on the HJ-distance

2.1. Population HJ-distance

Let x_t denote a vector of payoffs of n test assets at the end of period t and q_{t-1} be the corresponding costs of these n assets at the end of period $t-1$ with $E[q_{t-1}] \neq 0_n$.[3] This setup can accommodate both gross and excess returns on test assets as well as payoffs of trading strategies that are based on time-varying information. In addition, we assume that $U = E[x_t x_t']$ is nonsingular so that none of the test assets is redundant.

Let m_t represent an admissible SDF at time t and let $\mathcal{M}$ be the set of all admissible SDFs. An SDF m_t is admissible if it prices the test assets correctly, i.e.,[4]

$$E[x_t m_t] = E[q_{t-1}]. \tag{1}$$

Suppose that $y_t(\gamma)$ is a candidate SDF at time t that depends on a k-vector of unknown parameters $\gamma \in \Gamma$, where Γ is the parameter space of γ. An asset pricing model is correctly specified if there exists a $\gamma \in \Gamma$ such that $y_t(\gamma) \in \mathcal{M}$. The model is misspecified if $y_t(\gamma) \notin \mathcal{M}$ for all $\gamma \in \Gamma$. When the asset pricing model is misspecified, we are interested in measuring the degree of model misspecification. HJ suggest using

$$\delta = \min_{\gamma \in \Gamma} \min_{m_t \in \mathcal{M}} \left(E[(y_t(\gamma) - m_t)^2]\right)^{\frac{1}{2}} \tag{2}$$

as a misspecification measure of $y_t(\gamma)$. We refer to δ as the HJ-distance measure.

Instead of solving the above primal problem to obtain δ, HJ suggest that it is sometimes more convenient to solve the following dual problem:

$$\delta^2 = \min_{\gamma \in \Gamma} \max_{\lambda \in \Re^n} E[y_t(\gamma)^2 - (y_t(\gamma) - \lambda' x_t)^2 - 2\lambda' q_{t-1}], \tag{3}$$

where λ is an n-vector of Lagrange multipliers.

Let $\theta = [\gamma', \lambda']'$ and denote by $\theta^* = [\gamma^{*\prime}, \lambda^{*\prime}]'$ the pseudo-true value that solves the population dual problem in (3):

$$\theta^* = \arg \min_{\gamma \in \Gamma} \max_{\lambda \in \Re^n} E[\phi_t(\theta)], \tag{4}$$

where $\phi_t(\theta) \equiv y_t(\gamma)^2 - m_t(\theta)^2 - 2\lambda' q_{t-1}$ and $m_t(\theta) \equiv y_t(\gamma) - \lambda' x_t$. Note that $y_t(\gamma^*)$ prices the n test assets correctly if the vector of pricing errors is zero, i.e.,

$$e(\gamma^*) = E[x_t y_t(\gamma^*) - q_{t-1}] = 0_n. \tag{5}$$

In this case, $y_t(\gamma^*) \in \mathcal{M}$, $\lambda^* = 0_n$ and we refer to γ^* as the true value.[5]

[3] When $E[q_{t-1}] = 0_n$, the mean of the SDF cannot be identified and researchers have to choose some normalization of the SDF (see, for example, Kan and Robotti, 2008).

[4] Strictly speaking, the set of admissible SDFs should be defined in terms of conditional expectations. In this paper, we use an unconditional version of the fundamental pricing equation. This, in principle, could be justified by incorporating conditioning information through scaled payoffs (see, for example, Section 8.1 in Cochrane, 2005).

[5] The optimization problem in (4) bears strong resemblance to the structure of the Euclidean likelihood problem defined as $\min_\gamma \max_\lambda E[\mathbf{h}(\lambda' e(\gamma))]$ with $\mathbf{h}(\varsigma) = -\frac{1}{2}\varsigma^2 - \varsigma$. Other choices of $\mathbf{h}(\varsigma)$ give rise to some popular members of the class of generalized empirical likelihood (GEL) estimators. See Almeida and Garcia (2012) for further discussion of the class of GEL estimators in the context of asset pricing models. While the analysis in this paper can be easily extended to GEL estimators, we choose to present our main results for the HJ-distance measure given its popularity in empirical asset pricing, nice economic (maximum pricing error) interpretation and computational simplicity (closed-form solution for the Lagrange multipliers).