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Powerful tests for structural changes in volatility[☆]

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ABSTRACT

Detecting structural changes in volatility is important for understanding volatility dynamics and stylized facts observed for financial returns such as volatility persistence. We propose modified CUSUM and LM tests that are built on a robust estimator of the long-run variance of squared series. We establish conditions under which the new tests have standard null distributions and diverge faster than standard tests under the alternative. The theory allows smooth and abrupt structural changes that can be small. The smoothing parameter is automatically selected such that the proposed test has good finite-sample size and meanwhile achieves decent power gain.

1. Introduction

Structural change in volatility has attracted increasing attention from both theoretical and empirical researchers. Part of the driving force is the long-standing recognition that many stylized facts for the asset return volatility, such as long-range dependence or IGARCH effects, can be well explained by non-constant unconditional variances since Diebold (1986) and Lamoureux and Lastrapes (1990). More recently, Mikosch and Stărică (2004), Hillebrand (2005), and Perron and Qu (2010) provided further theoretical and empirical support for volatility models that can accommodate structural changes. Shifts in levels can have an impact on volatility forecast and financial risk management (Rapach and Strauss, 2008; Pettenuzzo and Timmermann, 2011). Smooth changes in the unconditional variance are also empirically relevant. Engle and Rangel (2008) and Engle et al. (2009) interpreted them as a low-frequency component of volatility that reflects the long-run dynamics of the volatility process. They proposed models of volatility components (see also Engle and Lee, 1999) and related the stock market volatility to macroeconomic fundamentals.

One of the most widely used tests for structural change in volatility is based on the cumulative sum (CUSUM) of squared series. It is especially useful when no prior knowledge of the type of structural change is available. Numerous researchers have proposed and empirically implemented versions of the CUSUM test; see Inclán and Tiao (1994), Loretan and Phillips (1994), Aggarwal et al. (1999), Kokoszka and Leipus (1999, 2000), Lee and Park (2001), Andreou and Ghysels (2002), Sansó et al. (2004), Malik et al. (2005), Deng and Perron (2008b), Rapach and Strauss (2008), Cavaliere and Taylor (2008), and Xu (2008), among others. They differ in how they accommodate stylized features in asset returns like non-normality and serial dependence. Inclán and Tiao (1994) designed their test for independently and identically distributed (i.i.d.) normal samples, and other researchers then extended this test and made it more robust. We show that extant CUSUM-based tests, robust or not, are constructed without considering any explicit alternative hypotheses. This opens the tests to criticism for having low power in practice, even though they are consistent against a broad range of alternatives. In this paper, we specify a nonparametric alternative that allows for smooth or abrupt changes in volatility without compromising the omnipresent diagnostic ability of the CUSUM-based test. We then propose a modified test that builds in appropriate information through nonparametric estimation of the unknown volatility function. We

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also consider an analogously modified test based on the Lagrange multiplier (LM) principle. The new tests we develop are simple to use and reduce to standard tests for particular choices of tuning parameters. We provide limit theory and establish conditions under which these new tests have standard null distributions and diverge at least as fast as standard ones under the alternative. We also address the practical issue of how to determine a bandwidth that delivers a test with good size and power.

This research fits into a larger literature on the detection of structural change (Perron, 2006) and is especially related to the case of changing means. One of central issues that stimulates much research is that tests based on CUSUM and other tests that allow for serial correlation may have non-monotonic power functions, and, in particular, have low power against large breaks (Vogelsang, 1999; Deng and Perron, 2008a). Altissimo and Corradi (2003), Crainiceanu and Vogelsang (2007), and Juhl and Xiao (2005, 2009) recently tackled this issue. In particular, my approach is analogous to that of Juhl and Xiao (2009), who also exercised nonparametric estimation of the time-changing function under the alternative. Their theoretical justification is not directly extendable, and, as we show, their key assumption of homoscedasticity is not satisfied here. Researchers recognized they must use caution when extending properties of the model of the first moment to that of the second moment. One example is that the square of a series that follows an integrated GARCH model has exponentially decaying autocorrelations, which seems counter-intuitive in view of the unit-root-in-squares representation of the model (Ding and Granger, 1996; Davidson, 2004). Using a similar model to the one we use in this paper, Xu (2011) showed that the CUSUM test for volatility breaks has monotonic power in contrast to the test for mean breaks.

The setting in this paper is nonparametric without modeling levels of the time series or types of structural breaks. Testing for volatility breaks in a GARCH-type parametric framework has been considered by Chu (1995) and Lundbergh and Teräsvirta (2002), among others. In contrast to our historical or retrospective approach, some of the literature focuses on sequential testing or real-time/on-line detection of change points in volatility that can be used sequentially as new data arrive; see Chu et al. (1996), Berkes et al. (2004), and Horváth et al. (2006).

The paper is organized as follows. After introducing the model and standard tests in Section 2, new tests based on CUSUM and LM principles via nonparametric estimation of the unknown volatility function are proposed in Sections 3 and 4. The limit theory is provided and implementation issues are also discussed. In Section 5, we present Monte Carlo results and an empirical application. Mathematical proofs are contained in an Appendix.

2. The model and the standard test

Consider the model for a sequence u_t (e.g. log returns):

$$u_t = \sigma_t \varepsilon_t, \tag{1}$$

where σ_t is a deterministic function of t and ε_t satisfies $E\varepsilon_t = 0$ and $E\varepsilon_t^2 = 1$. In (1), σ_t represents nonstationary unconditional variances and ε_t accounts for potential conditional heteroscedasticity. The variance process σ_t^2 satisfies $\sigma_t^2 = \sigma^2(t/n)$, where n is the sample size and $\sigma^2(\cdot)$ is a nonstochastic càdlàg (right continuous with left limits) function on $(0, 1]$ with a finite number of points of discontinuity. We also assume that $\sigma^2(\cdot) > 0$ and that it is at least twice differentiable except at the points of discontinuity with the second derivative function satisfying a first-order Lipschitz condition piecewise.

Model (1) is quite general, including regime-dependent GARCH models. The spline-GARCH model proposed by Engle and Rangel (2008) specifies σ_t as an exponential spline function of t to capture the low-frequency component of the equity volatility, and ε_t as a mean-reverting GARCH to capture the high-frequency component.[1]

The hypotheses of interest are

$$H_0 : \sigma_t^2 = \sigma_0^2$$
$$H_A : \sigma_t^2 = \sigma_0^2 + g(t/n)n^b,$$

where $b \le 0$ and $g(\cdot)$ is a bounded function satisfying assumptions imposed on $\sigma^2(\cdot)$. Under H_0, σ_t^2 is time invariant. Under H_A, σ_t^2 is time dependent, but the degree of nonstationarity can be small depending on the index b. For example, if $\sigma^2(\cdot)$ is a step function (as in DGP 1 in Section 5), the jump size is proportional to n^b. When $b = 0$, H_A is the usual fixed alternative.

Let $D_M = \sum_{t=1}^{M} u_t^2 - (M/n)\sum_{t=1}^{n} u_t^2$ for $M = 1, \ldots, n$. The CUSUM test is defined as $Q = \max_{1 \le M \le n} Q(M)$, where $Q(M) = n^{-1/2}|D_M|/\overline{\omega}$ and $\overline{\omega}^2$ is the estimator of the long-run variance (LRV) of $u_t^2 - \sigma_t^2$ under the null, i.e. $\overline{\omega}^2 = \overline{\gamma}_0 + 2\sum_{l=1}^{n-1} k(l/m)\overline{\gamma}_l$, with $\overline{\gamma}_l = n^{-1}\sum_{t=l+1}^{n}(u_t^2 - \widehat{\sigma}^2)(u_{t-l}^2 - \widehat{\sigma}^2)$, for $l = 0, 1, \ldots, n-1$, and $\widehat{\sigma}^2 = \sum_{t=1}^{n} u_t^2/n$. Here, $k(\cdot)$ is a kernel function assigning weights to autocovariance estimates (Newey and West, 1987; Andrews, 1991) and m is a truncation parameter. The test based on the statistic Q has been considered by Loretan and Phillips (1994), Lee and Park (2001), Andreou and Ghysels (2002), Sansó et al. (2004), Malik et al. (2005), Deng and Perron (2008b), Rapach and Strauss (2008), and Cavaliere and Taylor (2008), among others. The null distribution of Q follows the supremum of a Brownian bridge; e.g. see Deng and Perron (2008b) for a treatment under similar assumptions. We focus on the behavior of Q under H_A in this section.

The test Q is closely related to the CUSUM test for structural change in mean. Rewrite model (1) as

$$u_t^2 = \sigma_t^2 + e_t, \tag{2}$$

where $e_t = \sigma_t^2(\varepsilon_t^2 - 1)$. Under H_0, (2) is simply a location model for u_t^2 with homoscedastic and serially correlated errors. So the problem of interest is translated to testing for changing mean in u_t^2. Indeed, Q is identical to the OLS residual-based CUSUM statistic (Ploberger and Krämer, 1992) applied to u_t^2. Such an equivalence, however, comes with a caveat. Under the alternative, structural change occurs not only in the mean of u_t^2 but also in its variance, so extant results on the power of the CUSUM test in the context of a changing mean do not directly apply here unless heteroscedasticity is allowed. For example, the well-known non-monotonic-power issue for the CUSUM test (Vogelsang, 1999) does not apply to the volatility break test because the simultaneous (large) break in both the variance and the fourth moment prohibits the automatic procedure selecting excessive lags in the LRV estimator, the main source of potential non-monotonic power

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[1] The empirical evidence of Engle and Rangel (2008) shows that the long-term volatilities of macroeconomic fundamentals, like GDP, interest rates and inflation, and market sizes may be causes of low-frequency market volatility. In a related paper, Engle et al. (2009) proposed models under a mixed data sampling scheme and directly incorporated macroeconomic variables that are usually sampled at a low frequency. Both of these studies find that consideration of the low-frequency component at least pays off in long-horizon forecasting of stock market volatility. In the models of Engle et al. (2009), σ_t is a stochastic process and does not fit directly in our framework. Extension to allow for a long-run component generated by random sources is left for future investigation.