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Functional coefficient regression models with time trend[☆]

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ABSTRACT

We consider the problem of estimating a varying coefficient regression model when regressors include a time trend. We show that the commonly used local constant kernel estimation method leads to an inconsistent estimation result, while a local polynomial estimator yields a consistent estimation result. We establish the asymptotic normality result for the proposed estimator. We also provide asymptotic analysis of the data-driven (least squares cross validation) method of selecting the smoothing parameters. In addition, we consider a partially linear time trend model and establish the asymptotic distribution of our proposed estimator. Two test statistics are proposed to test the null hypotheses of a linear and of a partially linear time trend models. Simulations are reported to examine the finite sample performances of the proposed estimators and the test statistics.

1. Introduction

Trending is an important topic in economics such as trends in productivity growth and economic growth. As Krugman (1995) commented: "Productivity growth is the single most important factor affecting our economic well being…", and as noted in Andrews and McDermott (1995), "it is clear that most macroeconomic variables and many financial variables exhibit trends". Most of the existing researches adopt models with a linear trend or a stochastic trend with a constant drift. When a constant drift is included in a stochastic trend model, this results in a model with the time trend variable having a constant coefficient while the disturbance has a variance that grows over time. In this paper we take an alternative approach to model the trend behavior. In our model, time trend has a stochastic coefficient while we also consider a partially linear specification where the coefficient of the time trend variable is a constant, the coefficients of other variables vary with a stationary covariate. Specifically, we consider a varying coefficient model of the form: $Y_t = X_{1t}^{\top}\beta_1(Z_t) + t\beta_2(Z_t) + u_t$, where $\beta_j(z)$, $j = 1, 2$, is a smooth function of z. A functional coefficient time trend model provides much flexibility and may help to identify the sources of changes. There is a literature that views the trend as deterministic trends with (multiple) breaks, e.g. see Perron (1989) and Zivot and Andrews (1992). The deterministic trend with multiple break points are closely related to our model. Our model is also related to a regime switching model with time trend and with an observable state variable Z_t such that $Y_t = X_{1t}^{\top}\beta_1 + t\beta_2 + u_t$ if $Z_t < c$, and $Y_t = X_{1t}^{\top}\alpha_1 + t\alpha_2 + u_t$ if $Z_t \geq c$, where c is a constant and the state variable Z_t determines the regimes. A regime switching model allows for discrete jump for the β coefficient, but within each regime the coefficient stays constant. In our framework the coefficient $\beta(\cdot)$ is allowed to vary smoothly with respect to a relevant stationary covariate. In addition, our model contains the traditional linear time trend model, or a more general partially linear time trend model, as special cases.

There are other related works on nonlinear time trend models. Andrews and McDermott (1995) studied the nonlinear parametric econometric models with deterministically trending variables. Robinson (1989) was the first to study a nonparametric time-varying coefficient model in which he modeled regression coefficients as an unspecified smooth function of time. Cai (2007) extended Robinson's model to the case with serially correlated disturbance terms and applied the method to US stock market data to estimate a varying coefficient CAPM, and found the β-coefficient was indeed time varying and exhibited interesting patterns. In this paper we follow a more traditional approach

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by introducing the time trend as a regressor but its coefficient can change smoothly over some relevant stationary covariate. We find that the asymptotic results are somewhat different from conventional nonparametric regression models with stationary ($I(0)$) or unit-root non-stationary ($I(1)$) regressors. For example, we show that surprisingly, the popularly used local constant kernel estimation method leads to an inconsistent estimation result, while a local polynomial method can be used to consistently estimate the model.

Our approach extends the existing functional coefficient regression model literature a step further to include a time trend as a regressor. Recently, the varying coefficient models have attracted much attention among econometricians and statisticians. One attractive feature of this model is its ability to capture the nonlinearity of the data and its capacity to ameliorate the so-called "curse of dimensionality". Another advantage is its interpretability. It provides simple low dimension curves (often univariate curves) which describe the marginal effects of explanatory variables on the dependent variable. Cai et al. (2000), Fan and Zhang (2000), Zhang et al. (2002), Cai (2007), Cai et al. (2009) and Xiao (2009) are among the recent works using this model to deal with stationary and non-stationary time series data.[1]

The paper is organized as follows. In Section 2, we consider a varying coefficient model with a time trend and some stationary regressors. Asymptotic theory of a local polynomial estimator is derived. We also provide asymptotic analysis of the least squares cross validation selected smoothing parameter, and study a partially linear varying coefficient model. In Section 3, we propose two test statistics for testing a linear and a partially linear varying coefficient models. We report simulation results in Section 4 to examine the finite sample performances of our proposed estimators and the test statistics. The proofs are relegated into three appendices with the supplementary Appendix C available upon request.

2. A time trend varying coefficient model

2.1. Local polynomial estimation method and asymptotic results

We consider the following time trend varying coefficient model.

$$Y_t = X_t^\top \beta(Z_t) + u_t = X_{1t}^\top \beta_1(Z_t) + t\,\beta_2(Z_t) + u_t, \tag{2.1}$$

where Z_t is a scalar, X_{1t} is a $(d-1)\times 1$ vector of stationary regressors ($d \geq 2$), $\beta_1(\cdot)$ is a $(d-1)\times 1$ vector-valued smooth coefficient function associated with X_{1t}.

We will use a qth-order ($q \geq 1$) local polynomial method to estimate model (2.1). Let $\mathcal{B}(z) = (\beta_1(z)^\top, \beta_2(z), \beta_2^{(1)}(z), \ldots, \beta_2^{(q)}(z))^\top$ with $\beta_2^{(j)}(z) = \frac{d^j\beta_2(z)}{dz^j}$, $j = 1, \ldots, q$, and define $\tilde{X}_t^\top = (X_{1t}^\top, t, t(Z_t - z), \frac{1}{2!}t(Z_t - z)^2, \ldots, \frac{1}{q!}t(Z_t - z)^q)$. Then the qth order local polynomial estimator $\widehat{\mathcal{B}}(z) = (\hat\beta_1(z)^\top, \hat\beta_2(z), \hat\beta_2^{(1)}(z), \ldots, \hat\beta_2^{(q)}(z))^\top$ is given by

$$\widehat{\mathcal{B}}(z) = \underset{\mathcal{B}}{\operatorname{argmin}} \sum_{t=1}^{n} [Y_t - \tilde{X}_t^\top \mathcal{B}]^2 K_{h,z_t z},$$

where $K_{h,z_t z} = h^{-1}K((Z_t - z)/h)$. $K(\cdot)$ is the kernel function and h is the smoothing parameter.

Note that we use local constant approximation for $\beta_1(\cdot)$ and local polynomial approximation for $\beta_2(\cdot)$. Of course we could also use local polynomial approximation for $\beta_1(\cdot)$. However, this will introduce $(d-1)q$ extra parameters to be estimated (the derivatives of the $(d-1)\times 1$ vector-valued function $\beta_1(\cdot)$). Since the nonparametric kernel method only uses local data (data close to z) when estimating $\beta(z)$, an estimation method with too many parameters may severely limit the usefulness of the nonparametric estimation method in practice.

It is easy to show that $\hat{\mathcal{B}}(z)$ has the following close form expression:

$$\begin{aligned}\hat{\mathcal{B}}(z) &= \left[\sum_{t=1}^n \tilde{X}_t\tilde{X}_t^\top K_{h,z_t z}\right]^{-1} \sum_{t=1}^n \tilde{X}_t (X_{1t}^\top \beta_1(Z_t) \\ &\quad + t\beta_2(Z_t) + u_t) K_{h,z_t z} \\ &= \mathcal{B}(z) + \left[\sum_{t=1}^n \tilde{X}_t\tilde{X}_t^\top K_{h,z_t z}\right]^{-1} \sum_{t=1}^n \tilde{X}_t \left[X_{1t}^\top(\beta_1(Z_t)\right. \\ &\quad \left. -\ \beta_1(z)) + t\left(\beta_2(Z_t) - \sum_{i=0}^q \frac{1}{i!}\beta_2^{(i)}(z)\,(Z_t - z)^i\right)\right] \\ &\quad \times K_{h,z_t z} + \left[\sum_{t=1}^n \tilde{X}_t\tilde{X}_t^\top K_{h,z_t z}\right]^{-1} \sum_{t=1}^n \tilde{X}_t u_t K_{h,z_t z}.\end{aligned} \tag{2.2}$$

The first $(d-1)$ elements of $\hat{\mathcal{B}}(z)$ estimate $\beta_1(z)$, and the remaining $(q+1)$ elements estimate $\beta_2(z)$ and its derivatives up to the order q. Let D_n be a $(d+q)\times(d+q)$ diagonal matrix defined by $D_n = \mathrm{Diag}(1, \ldots, 1, n, nh, \ldots, nh^q)$, $S_n(z) = D_n^{-1}[\frac{1}{n}\sum_{t=1}^n \tilde{X}_t\tilde{X}_t^\top K_{h,z_t z}]D_n^{-1}$, $L_{2n}(z) = \frac{1}{n}\sum_{t=1}^n D_n^{-1}\tilde{X}_t u_t K_{h,z_t z}$ and $L_{1n}(z) = \frac{1}{n}\sum_{t=1}^n D_n^{-1}\tilde{X}_t[X_{1t}^\top(\beta_1(Z_t) - \beta_1(z)) + t(\beta_2(Z_t) - \sum_{i=0}^q \frac{1}{i!}\beta_2^{(i)}(z)\,(Z_t - z)^i)]K_{h,z_t z}$. Then it is easy to show that

$$D_n[\hat{\mathcal{B}}(z) - \mathcal{B}(z)] = S_n(z)^{-1}(L_{1n}(z) + L_{2n}(z)).$$

Below we list some regularity conditions. We assume that $\{(X_{1t}^\top, Z_t, u_t)\}_{t=-\infty}^{+\infty}$ is a strictly stationary process. We use $C^l = C^l(\mathcal{D})$ to denote the space of functions that has a continuous lth derivative function on $\mathcal{D}$, where $\mathcal{D}$ is the support of Z_t.

Assumption 2.1. The coefficient function $\beta_1(\cdot) \in C^3$, $\beta_2(\cdot) \in C^{q+3}$, where $q \geq 1$ is a positive integer, $f(\cdot) \in C^2$ and $\sigma^2(x_1, \cdot) \in C^2$ for all x_1 in the support of X_{1t}, where $f(\cdot)$ is the density function of Z_t and $\sigma^2(x_1, z) = E(u_t^2 | X_{1t} = x_1, Z_t = z)$.

Assumption 2.2. a. The kernel function $K(\cdot)$ is a bounded and symmetric density function with a compact support $\mathcal{S}_K$. Also, $K(\cdot)$ satisfies the Lipschitz condition, that is, $|K(u) - K(v)| \leq C|u - v|$ for all $u, v \in \mathcal{S}_K$, where C is a positive constant.

b. $|g(u, v|x_0, x_1; l)| \leq M_1 < \infty$, for all $l \geq 1$, where $g(u, v|x_0, x_1; l)$ is the conditional density function of (Z_0, Z_l) given (X_{10}, X_{1l}), and $f(u|x) \leq M_2 < \infty$, where $f(u|x)$ is the conditional density function of Z_t given $X_{1t} = x$.

c. The process $\{(X_{1t}^\top, Z_t, u_t)\}$ is α-mixing with $\sum_{j=1}^\infty j^\gamma[\alpha(j)]^{1-2/\delta} < \infty$ for some $\delta > 2$ and $\gamma > 1 - 2/\delta$. Also, $E\|X_{1t}\|^{2\delta} < \infty$.

d. $E[u_0^2 + u_l^2 | Z_0 = z, X_{10} = x_0; Z_l = z', X_{1l} = x_1] \leq M_3 < \infty$, for all $l \geq 1$, $x_0, x_1 \in \mathbf{R}^{d-1}$, z and z' in a neighborhood of z_0.

e. There exists $\delta^* > \delta$, where δ is given in condition 2.2c, such that $E[|u|^{\delta^*} | Z_t = z, X_{1t} = x] \leq M_4 < \infty$, for all $x \in \mathbf{R}^{d-1}$ and z in a neighborhood of z_0, and $\alpha(n) = O(n^{-\theta^*})$, where $\theta^* \geq \delta\delta^*/\{2(\delta^* - \delta)\}$. Also, $E\|X_{1t}\|^{2\delta^*} < \infty$, and $n^{-(\delta/4-1/2)} h^{-(\delta/4+1/2-\delta/\delta^*)} = O(1)$.

f. As $n \to \infty$, $h \to 0$ and $nh \to \infty$. Further, there exists a sequence of positive integers s_n such that $s_n \to \infty$, $s_n = o((nh)^{1/2})$ and $(n/h)^{1/2}\alpha(s_n) \to 0$, as $n \to \infty$.

[1] For a variety of economic applications of varying coefficient models, see Mamuneas et al. (2006), Stengos and Zacharias (2006), among others.