Computers and Structures 160 (2015) 20–39

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

A time-domain finite element method for dynamic viscoelastic solution of layered-half-space responses under loading pulses

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ARTICLE INFO

Article history:
Received 23 August 2014
Accepted 30 July 2015
Available online 22 August 2015

Keywords:
Layered half-space
Dynamic response
Viscoelasticity
FE method
Computer program

ABSTRACT

This research develops a dynamic viscoelastic model, a Galerkin based time-domain finite element method, and computer program for simulating layered half-space responses under loading pulses. A combined Houbolt, central finite-difference (FD) and forward FD method is proposed for time discretization of acceleration and velocity to reduce time-step lengths. Compared to existing methods, the developed approach has advantages that it: (1) captures the coupled effects of material viscoelasticity, dynamics and system damping, which fosters understanding structural responses and material deformations; (2) is able to model temperature and space-dependent material properties. The model is implemented and validated for a multilayer pavement-soil structure.

1. Introduction

A half space is geometry of structure that has one side infinite or semi-infinite, and the layered half space is a structure with multi-layers sitting on the half space. A layered structure built on the soil foundation is a typical layered half-space, such as the multilayer pavements constructed on soil for highways, airport runways, and parking lots to sustain vehicle loadings. As shown in Fig. 1, a layered half-space of pavement-soil structure consists of four layers: asphalt concrete (AC) or Portland cement concrete as the surface course, base layer (e.g., aggregate and stabilized base), sub-base layer (e.g., gravel material), and soil foundation (semi-infinite half-space). The vehicle tire contact area could be approximated as a circular for analysis and design [1]. The plate loading tests are used to evaluate the structural capacity. In this test a circular plate is placed on the surface of the soil or layered structure and a mass is dropped to hit the plate, which produces a loading pulse within a short time period (e.g., 0.1 s or less) (see Fig. 1). Among these plate loading tests, the falling weight deflectometer (FWD) test is a popular one emulating the vehicle loading effect, where several geophones are also placed at various distances to record the deflection responses (see Fig. 1).

Understanding structural response of the layered half-space under external loading is a key for risk assessment (e.g., deformations under seismic loading) and structural design. Multiple models and computer methods have been developed or employed to simulate responses of deflection, stress, and strain for the (layered) half-space of pavement on soil foundation. These models and research progress are reviewed and summarized as follows:

(i) **Analytical approaches for half-space and layered half-space**. Boussinesq's solution [2] has been used to calculate responses of an elastic half-space (e.g., soil) under a static point loading. Lamb [3] might be the first researcher to develop a formulation for the surface motion of homogeneous elastic half-space under a point pulse. Extensive research has attempted to solve Lamb's problem, including the most recent publication by Kausel [4], who derived a complete set of exact explicit formulas for the suddenly applied point loads. Miller and Pursey [5] derived a solution for the harmonic uniform circular load applied to the elastic half-space.
Among analytical approaches, the most widely used methodology is the multilayer analysis program. The method is based on linear elasticity theory for an axisymmetric multilayer structure with semi-infinite half-space using Hankel transforms. Samples of well-known multilayer analysis programs include the ELSYM 5, which was originally developed by the University of California at Berkeley and later adapted to microcomputers by Kopperman et al. [6]. However, these multilayer analysis programs can account only for the static loading and linear elastic material property. Some materials such as the AC is a typical viscoelastic or viscoplastic material, AC is highly temperature and time dependent [7,8], and thus the linear elasticity

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http://dx.doi.org/10.1016/j.compstruc.2015.07.005
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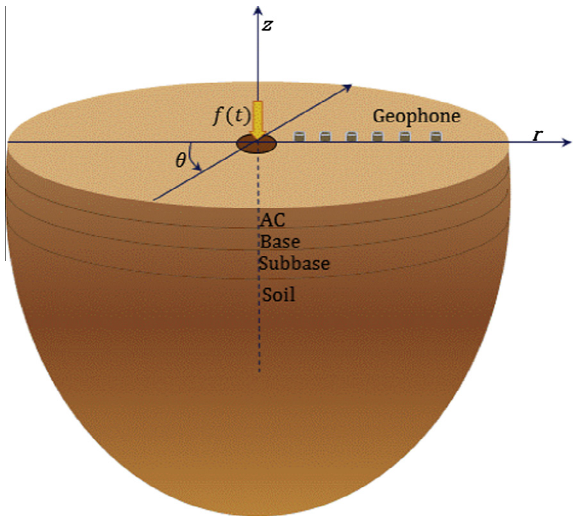

Fig. 1. A layered half-space of flexible pavement structure on soil under circular loading.

theory may not be accurate enough to capture the material behavior under the dynamic loading. Consequently, some researchers have extended the multilayer analytical methods by accounting for the material's viscoelastic behavior or dynamic loading effects or both. Hopman [9] and Kim [10] developed the viscoelastic solutions of the multilayer structure under a static loading mode. Most recently Lee [11] developed a dynamic viscoelastic analysis program called ViscoWave based on the Laplace and Hankel transforms, as extended from the layered linear elastic approach. However, the damping effect has not been taken into account [11]. Fig. 2 briefly illustrates these developments of analytical approaches for solutions of the half-space and layered half-space.

(ii) **Numerical methods**. Different numerical algorithms could be used for response modeling of the layered half-space structure including the finite element (FE) method, spectral element method [12], and boundary element method [13], hybrid element method [14]. In comparison to the multilayer analysis programs, the FE modeling could account for more complex conditions including the variable boundary and loading conditions, and advanced material models. Dave et al. [15] developed a functionally graded FE model to account for the non-homogeneous viscoelastic material property under a static loading pattern. Al-Qadi et al. [16] modeled the creep behavior of AC material under a static loading history using the FE method using ABAQUS.

(iii) **Combined and other methods**. Ayadi et al. [17] developed a dynamic semi-analytical and FE model to simulate pavement deflections under the FWD loading. Kausel and Park [18] derived the dynamic elastic responses of the layered half-space in time domain based on the thin layer method. Sun et al. [19] proposed a high-order thin layer method for modeling viscoelastic wave propagation in the stratified media. Most recently in 2013, the University of Nevada at Reno developed the 3-D Move Analysis Software [20] based on the finite layer method using Fourier transforms for each layer. The 3-D vehicle contact stress distributions are generated from the moving loading [21]. The material model input for the AC layer is the mathematical sigmoidal function of dynamic modulus, but phase angle is a user input option to determine a damming ratio for dynamic analysis. Chaillat and Bonnet [22] proposed a multiple formulation for the solution of elastic dynamic half-space of soil using Green's tensor. Timonin [23] proposed a finite-layer method for the linear elastic stress–strain analysis of layered composites, where each particular layer is considered a constituent of the entire laminate structure; the nonlinear geometry was also considered.

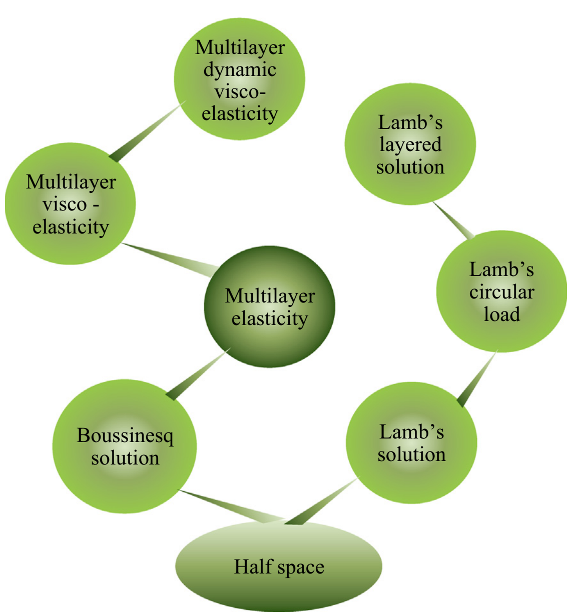

Fig. 2. Analytical solutions of (layered) half space.

One shall note that analytical and numerical methods for modeling viscoelastic and/or dynamic behaviors of a uniform solid media have also been developed (not for composites or layered systems). Some examples are reviewed as follows. Oguibe and Webb [24] studied the large deformations of the multilayer cantilever beams under dynamic loading. Makris [25] examined a convolution integral analytical solution for the time domain analysis of the viscoelastic model of a soil body. Guénette and Fortin [26] proposed a mixed FE method to compute the viscoelastic flow of benchmark problems. Shaw and Whiteman [27,28] proposed a space–time Galerkin FE discretization method for solving the linear quasistatic compressible viscoelasticity problem based on the elliptic partial differential equation. Kim and Paulino [29] proposed a framework for a generalized isoparametric formulation using the graded FE method, which can possess spatially varying material properties. Qin et al. [30] developed an algorithm to simulate the static viscoelastic response of a single body under the moving loading pattern.

2. Research motivation and significance

First, most existing computer methods and programs of layered half-space, such as those mentioned above, have primarily considered limited aspects – either the dynamic effect, the elastic or viscoelastic behavior of material, or both but without considering the damping effect (e.g., the analytical solution developed by Lee [11]). Therefore, a more comprehensive model and computer method considering the dynamic loading effect, material's viscoelastic behavior, and the system damping effect may more accurately capture the coupled effects of loading and material behavior. Secondly, the in-service situations of the layered half-space are indeed more complex than theoretical assumptions often used in existing modeling studies, such as the assumed uniform temperature and