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Numerical limit analysis of steel-reinforced concrete walls and slabs

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ABSTRACT

A limit analysis based design methodology is hereafter proposed and applied for the peak load evaluation of steel-reinforced concrete large-scale prototypes of structural *walls* and *slabs*. The methodology makes use of nonstandard limit analysis and predicts the peak load multiplier of the analyzed structures by detecting an upper and a lower bound to it. Some useful hints on the collapse mechanism the structure will exhibit at its limit state is also attainable. To check the reliability of the numerically detected peak loads and failure modes a comparison with experimental laboratory findings, available for the large-scale specimens considered, is presented.

1. Research motivations and introduction

The present study follows a very recent paper by the authors [30], dealing with the possibility to predict the peak load and the failure mechanism of steel-reinforced concrete elements. Such a possibility, explored in [30] with reference to a standard benchmark on steel-reinforced concrete *beams* under bending, is investigated here with reference to reinforced concrete (RC) structures of practical and greater engineering interest, namely: *walls* and *slabs*. The addressed topic belongs to a wider ongoing research programme, started by the authors in [29] in the context of RC structures, but already applied with success to structural elements made by different constitutive materials, such as laminates of fiber reinforced polymers, see e.g. Pisano and Fuschi [26,27], Pisano et al. [28]. The proposed methodology, giving information at an ultimate (collapse) state of the structure in terms of peak load and collapse mechanism, can be viewed as a useful preliminary design tool also for RC-structures of large dimensions. If necessary, more accurate step-by-step analysis, able to follow the fracturing/damaging processes exhibited by RC-structures in the post-elastic regime, can be carried out. Such a deeper investigation can however be reserved *only* to confined zones or weaker structural elements detected by a much simpler limit analysis which, as shown hereafter, gives useful hints on the collapse mechanism and predicts, with good accuracy, the ultimate value of the loads acting on the located weaker members or parts.

The numerical analysis employed here is based, essentially, on the application of *non standard limit analysis theory* (Lubliner [21]). The peak load of a structure made by a non standard material as concrete (where non associativity is postulated to account for its dilatancy) can indeed be located between an upper and a lower bound to it. There are many examples of limit analysis in the realm of nonstandard materials, from the pioneering works of Drucker et al. [9] and Radenkovic [33], to studies concerning geotechnical problems, e.g. Sloan [37], Boulbibane and Ponter [5], or those specifically dealing with reinforced concrete, see e.g. Liman et al. [20], Larsen et al. [16]. A comprehensive and updated review of limit analysis methods can be found in Nielsen and Hoang [25] in the field of concrete plasticity or, in the wider context of the so-called Direct Methods, in the very recent book by Spiliopoulos and Weichert [38]. On the other hand, the application of plasticity based approaches to reinforced concrete structures, whose ductile behavior is assured by the presence of reinforcement, is witnessed by a number of contributions, see e.g. the monographs of Chen [7,8] and, again, Nielsen and Hoang [25] or the papers by Brisotto et al. [10], Roh et al. [34], Zhang and Li [41], Benkemoun et al. [3], Carrazedo et al. [6], Wu and Harvey [40], just to quote few very recent contributions on this research theme being the list far to be exhaustive.

There are two finite element (FE) based iterative procedures promoted by the authors for limit analysis of RC-structures, the linear matching method (LMM), see e.g. Ponter and Carter [31], Pisano et al. [29], and the elastic compensation method (ECM), see e.g. Mackenzie and Boyle [22], Pisano et al. [30]. In the above couples of references, the former paper is the one where (with reference to structures made of von Mises-type materials) the

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method was conceived and first proposed, the latter that where the method was rephrased to deal with reinforced concrete structural elements. In particular, in Pisano et al. [29] the LMM has been reformulated and adapted to a Menétrey–Willam-(M–W)-type yield criterion (Menétrey and Willam [24]) focusing all the theoretical aspects, the mathematical and geometrical details with reference to a 3D formulation in the Haigh–Westergaard coordinates. A few examples are presented there to show the applicability of the method to reinforced concrete simple elements. In Pisano et al. [30], while the effectiveness of the LMM is investigated by analyzing a standard benchmark on steel-reinforced concrete beams under bending (Bresler and Scordelis [4], Vecchio and Shim [39]), a revisited version of the ECM suitable for the M–W-type yield criterion is proposed. In the above paper the use of the two methods was applied for the first time to simple reinforced concrete structures, namely beams, showing the possibility to locate the real (experimentally detected) value of the peak load by computing an upper and a lower bound to it.

In the *present study*, skipping the theoretical details given in [29,30] to avoid repetition, the above mentioned numerical FE procedures are applied to compute the peak load as well as to predict the failure mechanisms of large-scale RC-prototypes of *walls* and *slabs*. The relevant experimental data, concerning tests carried out up to collapse and available for the analyzed specimens, have been used to validate the numerical predictions so facing real experimental findings. The following papers/reports have been considered: Lefas et al. [18], where structural walls were tested under combined action of a constant axial and a horizontal load monotonically increasing to failure; El Maaddawy and Soudki [11], where simply supported one-way RC-slabs were tested to failure under four-point bending; Sakka and Gilbert [35,36], where simply, continuous and corner supported square and rectangular slabs subjected to line or point loads increasing to failure were tested. Some of the results of these latter tests are also reported in more recent publications of the same Authors to which the Reader can refer [13,14].

Some information on the followed nonstandard limit analysis approach as well as on the LMM and ECM are given in the next Section 2 where the key ideas of the iterative FE numerical schemes are explained with the aid of two geometrical sketches. Details of the computational steps are given in Appendices A and B for completeness. Section 3 addresses the geometry, the material data, laboratory fixtures and the loading conditions of the analyzed RC-prototypes. The adopted mechanical model, FE meshes, modeling hypotheses are also expounded in this Section closing with a comparison between the obtained results and the experimental findings. Concluding remarks are given at closure in Section 4 also outlining possible future developments.

2. Key ideas of the numerical approach and computational schemes

The key point of the promoted *nonstandard limit analysis approach* (see e.g. [33]) is to encircle the yield surface of a given *nonstandard material* with two surfaces, precisely an outer and an inner surface and to compute, with reference to such surfaces (referred to two standard materials), an upper and a lower bound to the real collapse load multiplier pertaining to the nonstandard material structure under consideration. Concrete is herein modeled as a nonstandard material obeying to a M–W-type yield surface which can play the double role of inner and outer surface in the sense specified above. The M–W-type yield surface endowed with cap in compression is shown schematically in Fig. 1. Steel reinforcement, on the other hand, are considered of an infinitely elastic behavior. Their presence is taken into account only for what concern the confinement effect they exert on concrete. Such effect injects a ductile behavior on the RC-element that is it confers to

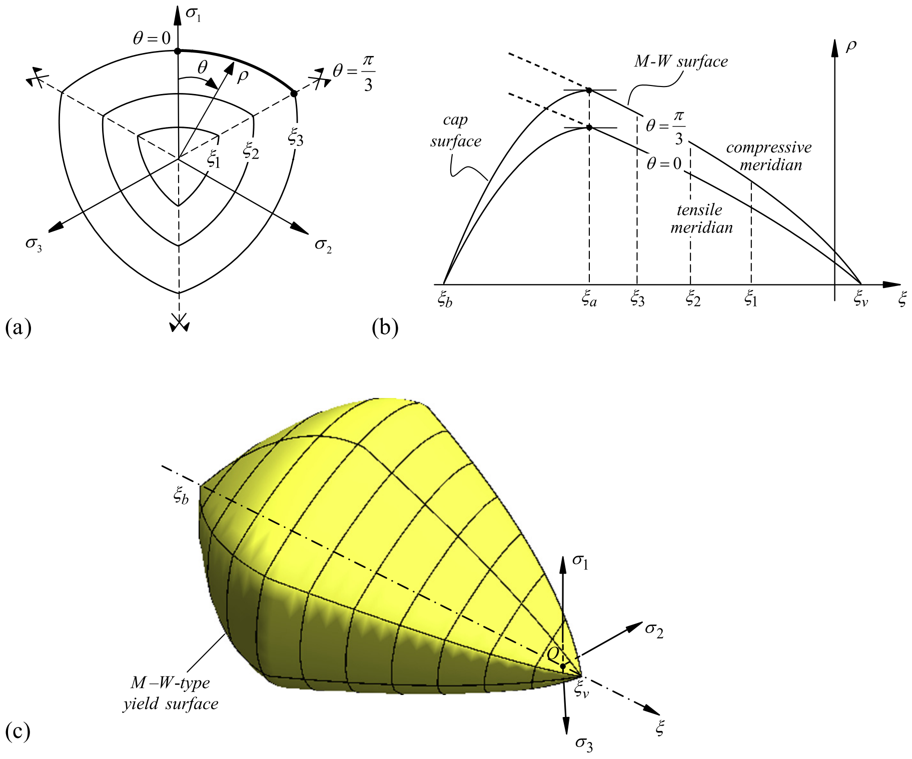

Fig. 1. Adopted Menétrey–Willam-type yield surface with cap: (a) deviatoric sections at three generic values of hydrostatic pressure; (b) tensile and compressive meridians in the Rendulic plane at $\theta = 0$ and $\theta = \pi/3$, respectively; and (c) 3D sketch in the principal stress space.