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Multilevel reduced-order computational model in structural dynamics for the low- and medium-frequency ranges

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ABSTRACT

This work deals with the dynamical analysis of complex structures composed of several structural levels and characterized by the presence of numerous local elastic modes intertwined with global modes, in the medium-frequency range as well as in the low-frequency range. For constructing the ROM, a family of global-displacements eigenvectors are calculated and are used instead of the classical elastic modes. Since it is also of importance to adapt the physical models (damping, level of uncertainties, etc.) to each one of the structural levels, a multilevel ROM is proposed. A validation is performed for an automobile complex structure.

1. Introduction

In linear structural dynamics, the frequency response functions (FRF) present isolated peaks at the resonance frequencies of associated global elastic modes, in the low-frequency (LF) range. In contrast, the high-frequency (HF) range presents rather smooth FRF due to the presence of a high and constant modal density. For complex structures, a medium-frequency (MF) range appears, for which the modal density exhibits large variations over this band [1,2]. The use of the first eigenmodes (or elastic modes, associated with the first eigenfrequencies) as a projection basis is particularly adapted to construct an accurate reduced-order model (ROM) of small dimension for analyzing the FRF in the low-frequency range [3–7]. Statistical energy methods (such as SEA [8]) are generally used for the high-frequency range analysis.

In this work, we are interested in complex structures characterized by the presence of numerous local elastic modes intertwined with global elastic modes, as soon as the low-frequency range. For instance, this unusual feature is related to (1) the presence of flexible parts attached to a stiff master part and (2) to the high complexity of the structure analyzed. For such a case, if the usual modal analysis is used, the ROM that is constructed can be of a very large dimension, due to the unusual presence of the numerous local elastic modes whose contributions are not necessarily significant for prediction of FRFs. This case is typically encountered in the low-frequency vibration of automobiles for which 5–10 global elastic modes can be intertwined with about 1000 local elastic modes in the frequency band $[0, 200]$ Hz, or for the dynamics of fuel assemblies in nuclear power plants, which can exhibit about 250 global elastic modes intertwined with about 50,000 local elastic modes in the frequency band $[0, 400]$ Hz.

To circumvent this difficulty, one solution would consist in using a modal sorting method. In general, such an approach is difficult to perform due to the fact that the elastic modes cannot always be defined either as global or as local elastic modes, since they can be combinations of both global and local displacements. Moreover, the contribution of the local displacements become predominant in the elastic modes when the frequency increases, which is such that the global displacements cannot easily be detected among the elastic modes.

A second way would consist in using substructuring techniques [9–13,6]. Such techniques firstly require to develop the computational model in substructures, and secondly that the stiff master part and the flexible parts be well identified. The specification of the work proposed is to develop a methodology that is adapted to a unique computational model without using substructuring data. In addition, for the complex structures we are interested in, it can be difficult to clearly separate the stiff part from the flexible parts.

In this paper, a new multilevel ROM is proposed for analyzing the dynamics of complex structures in the low- and medium-frequency ranges. This work is a continuation of previous research [14–16]. The general strategy proposed in this work relies on the

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separation of the displacements associated with each structural level (the different levels of stiffness of parts of the structure). The global level is the level associated with the stiff master part, from which the displacements are global. The local level is the level associated with the flexible parts that are attached to the master part, and from which the displacements can be local. In the master part, the global displacements are predominant while the local contributions are often negligible. The new multilevel ROM constructed allows for

(i) obtaining a small-dimension ROM for complex structures using a global-displacements basis,
(ii) adapting the physical models (damping, level of uncertainties, etc.) to the different levels of stiffness in the structure, using a unique multilevel ROM based on the use of several vector bases whose displacements correspond to the several distinct levels in the structure.

(i) The first objective of this paper is thus to construct a ROM of smaller dimension than the ROM obtained using classical modal analysis. The strategy used relies on the filtering of the local displacements so as to obtain a ROM whose associated reduction basis may be constituted of global-displacements vectors only. This filtering is performed by solving the usual generalized eigenvalue problem corresponding to the homogeneous conservative system, but in which the kinetic energy is approximated, while the elastic energy is kept exact. The filtering of the local displacements thus relies on the choice of approximation (reduced kinematics) for the kinetic energy. The convergence of the global-displacements ROM constructed is then controlled through the vector subspace associated with the reduced kinematics used.

(ii) The second objective concerns the construction of separated representations adapted to each type of structural level. It is based on the use of the methodology dedicated to the construction of a global-displacements ROM. The choice of vector subspace for the calculation of the kinetic energy allows the displacements associated with each structural level to be either considered as global or as local displacements.

We present a general method for the construction of the global-displacements ROM. The construction of a polynomial approximation for the kinetic energy, adapted to the filtering of the local displacements, is detailed. The formulation of the global-displacements ROM is given, and computational aspects are addressed. Based on this method, the construction of the proposed multilevel ROM is presented, using several vector bases whose displacements correspond to the several distinct structural levels. The theoretical part is followed by the presentation of an application devoted to an automobile for which the computational model used has been presented in [15].

2. Context and strategy

The reference computational dynamical model is introduced, followed by its classical reduction on elastic modes, allowing both the framework of the present work and the notation used to be introduced. Then, the general strategy proposed for constructing the global-displacements ROM is summarized.

Let Ω be the bounded domain of a tridimensional linear damped structure that is fixed on a part Γ_0 of its boundary $\partial\Omega$, such that there are no rigid body displacements. The structure is subjected to external loads on the other part Γ of $\partial\Omega$. We are interested in predicting the FRFs of the structure in the frequency band of analysis, $\mathcal{B} = [\omega_{\min}, \omega_{\max}]$, with $0 < \omega_{\min}$. The reference computational model is constructed using the finite element method [17,18]. For all ω in $\mathcal{B}$, the complex vector $\mathbb{U}(\omega)$ of the m DOFs of the finite element model, corresponding to the discretization of the displacement field, is the solution of the matrix equation

$$(-\omega^2\,[\mathbb{M}] + i\omega\,[\mathbb{D}] + [\mathbb{K}])\,\mathbb{U}(\omega) = \mathbb{F}(\omega), \tag{1}$$

where $[\mathbb{M}]$, $[\mathbb{D}]$, and $[\mathbb{K}]$ are the $(m \times m)$ positive-definite symmetric real mass, damping, and stiffness matrices. The complex vector $\mathbb{F}(\omega)$ is related to the discretization of the external forces. For the complex dynamical structures of interest, the number m of DOFs can be relatively high (a few millions or a dozen millions).

The first n eigenfrequencies $\{f_\alpha\}_\alpha$ and the associated elastic modes $\{\boldsymbol{\varphi}_\alpha\}_\alpha$ in $\mathbb{R}^m$ are obtained by solving the generalized eigenvalue problem,

$$[\mathbb{K}]\,\boldsymbol{\varphi}_\alpha = \lambda_\alpha\,[\mathbb{M}]\,\boldsymbol{\varphi}_\alpha, \tag{2}$$

where the positive eigenvalues $\{\lambda_\alpha\}_\alpha$ are such that $0 < \lambda_1 \leqslant \cdots \leqslant \lambda_n$, from which the eigenfrequencies $\{f_\alpha\}_\alpha$ are given by $f_\alpha = \sqrt{\lambda_\alpha}/2\pi$. Let $[\Phi] = [\boldsymbol{\varphi}_1 \; \cdots \; \boldsymbol{\varphi}_n]$ be the $(m \times n)$ real matrix such that $[\Phi]^T\,[\mathbb{K}]\,[\Phi] = [\Lambda]$ and $[\Phi]^T\,[\mathbb{M}]\,[\Phi] = [I_n]$, with $[\Lambda]$ the diagonal matrix of the first n eigenvalues. The classical modal analysis method consists in writing, for all ω in $\mathcal{B}$ and with $n \ll m$,

$$\mathbb{U}(\omega) \simeq \mathbb{U}^{(n)}(\omega) = \sum_{\alpha=1}^{n} q_\alpha(\omega)\,\boldsymbol{\varphi}_\alpha = [\Phi]\,\mathbf{q}(\omega), \tag{3}$$

in which the n-dimensional complex vector $\mathbf{q}(\omega)$ is a vector of generalized coordinates. The generalized damping matrix $[\mathcal{D}]$ is such that $[\mathcal{D}] = [\Phi]^T\,[\mathbb{D}]\,[\Phi]$. Introducing the generalized force $\boldsymbol{\mathcal{F}}(\omega) = [\Phi]^T\mathbb{F}(\omega)$, the classical ROM associated with Eq. (3) is written as

$$(-\omega^2\,[I_n] + i\omega\,[\mathcal{D}] + [\Lambda])\,\mathbf{q}(\omega) = \boldsymbol{\mathcal{F}}(\omega). \tag{4}$$

For the case of a structure that exhibits numerous local elastic modes, the ROM proposed is constructed by using a basis of a global-displacements space, instead of using all the elastic modes that are present in frequency band $\mathcal{B}$. Let $\mathcal{S}_{\text{glob}}$ be the global-displacements space spanned by some eigenvectors $\{\boldsymbol{\psi}_\alpha\}_\alpha$ of the following generalized eigenvalue problem,

$$[\mathbb{K}]\,\boldsymbol{\psi}_\alpha = \sigma_\alpha\,[\mathbb{M}_{\mathcal{A}_g}]\,\boldsymbol{\psi}_\alpha, \tag{5}$$

corresponding to the homogenous conservative system for which the kinetic energy is approximated while the elastic energy is kept exact. In Eq. (5), σ_α is the positive eigenvalue associated with $\boldsymbol{\psi}_\alpha$ and $[\mathbb{M}_{\mathcal{A}_g}]$ is the modified mass matrix that depends on the approximation subspace $\mathcal{A}_g$ associated with a choice of reduced kinematics for the kinetic energy. In previous work [14], the domain of the structure is partitioned into N_s subdomains, $\Omega_1, \ldots, \Omega_{N_s}$, and the reduced kinematics is constructed in choosing the displacement field as a constant in each subdomain. In such a case, only $3N_s$ global eigenvectors, $\{\boldsymbol{\psi}_\alpha\}_\alpha$, associated with finite eigenvalues, $\{\sigma_\alpha\}_\alpha$, can be obtained because there are only $3N_s$ generalized DOFs constituting the reduced kinematics for the mass matrix (3 translations per subdomain). The characteristic dimension of the subdomains allows for controlling the level of filtering of local displacements. In this case, for the continuous formulation, a projection operator h^r of the displacement field $\mathbf{u}$ onto the subspace of constant functions by subdomain is introduced, such that, for all $\mathbf{x}$ in Ω,

$$\{h^r(\mathbf{u})\}(\mathbf{x}) = \sum_{j=1}^{N_s} \mathbb{1}_{\Omega_j}(\mathbf{x})\,\frac{1}{m_j}\int_{\Omega_j} \rho(\mathbf{x}')\mathbf{u}(\mathbf{x}')d\mathbf{x}', \tag{6}$$

in which $\mathbb{1}_{\Omega_j}(\mathbf{x}) = 1$ if $\mathbf{x} \in \Omega_j$ and is zero otherwise, where $m_j = \int_{\Omega_j} \rho(\mathbf{x})d\mathbf{x}$ is the mass of subdomain Ω_j, and where ρ is the mass density. The finite element discretization $[H^r]$ of h^r is used for obtaining the reduced-kinematics mass matrix